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DETERMINATION OF DECAY COEFFICIENTS FOR COMBUSTORS WITH ACOUSTIC ABSORBERS

by

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16. Abstract

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Foreward

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ABSTRACT

An analytical technique for the calculation of linear decay coefficients in combustors with acoustic absorbers is presented. Tuned circumferential slot acoustic absorbers are designed for the first three transverse modes of oscillation and decay coefficients for these absorbers are found as a function of backing distance for seven different chamber configurations. The effectiveness of the absorbers for off design values of the combustion response and acoustic mode is also investigated. Results indicate that for tuned absorbers the decay coefficient increases approximately as the cube of the backing distance. For most off design situations the absorber still provides a damping effect. However, if an absorber designed for some higher mode of oscillation is used to damp lower mode oscillations a driving effect is frequently found.

Nomenclature

a	-	speed of sound
B	-	absorber backing distance
B_1, B_2	-	quantities defined after Equation (A-3)
C	-	quantity defined after Equation (A-3)
D	-	decay coefficient
D_{eff}	-	effective decay coefficient
F_1, F_2	-	functions defined after Equation (A-4)
G	-	Green's function satisfying Equation (A-2)
G_N	-	modified Green's function defined after Equation (A-2)
H_N	-	function defined after Equation (A-4)
i	-	unit imaginary = $\sqrt{-1}$
Im( )	-	imaginary part of ( )
j	-	positive integer
J_m	-	Bessel Function of the first kind of order m
K	-	absorber impedance
ℓ	-	positive integer
L	-	chamber length
L_a	-	absorber aperture length
ℓ_{eff}	-	effective absorber aperture length
M	-	Mach number
m	-	positive integer
n	-	positive integer or interaction index
$\vec{n}$	-	outward unit normal vector
p	-	pressure
$\vec{q}$	-	velocity vector

R	-	chamber radius
r	-	radial length
$\vec{r}$	-	position vector
$\vec{r}_o$	-	source position vector
Re( )	-	real part of ( )
S	-	surface area of chamber
S_c	-	surface area of absorber slot
S_N	-	surface area of nozzle exit plane
t	-	time
u	-	axial velocity
V	-	chamber volume
v	-	radial velocity
w	-	circumferential velocity
W_A	-	absorber slot width
z	-	axial coordinate
α	-	absorption coefficient
β_a	-	acoustic admittance
γ	-	ratio of specific heats
η_N	-	acoustic eigenvalue for mode N
θ	-	circumferential coordinate
λ	-	imaginary part of complex frequency
$\lambda_{\ell m}$	-	root of $J_m'(\lambda_{\ell m}) = 0$
$\Lambda_{\ell mn}$	-	normalizing constant defined after Equation (A-2)
$\mu_{\ell n}^{(j)}$	-	matrix discussed after Equation (A-4)

ρ - density

τ - sensitive time lag

ϕ - perturbation velocity potential

ψ - normalizing constant defined after Equation (A-3)

Ω_N - acoustic eigenfunction for mode N

ω - complex frequency of oscillation

Subscripts

A - absorber aperture quantity

C - quantity evaluated on chamber cylindrical periphery

i - injector quantity

N - nozzle quantity

o - mean value

Superscripts

' - perturbation quantity or derivative with respect to argument

* - dimensional quantity

~ - chamber quantity with no absorber present

^ - designates particular mode integers chosen

(j) - approximation of order j

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SUMMARY

The effectiveness of a certain type of acoustic absorber as a damping device in seven different rocket motors has been evaluated analytically.

The type of absorber considered in this study was an annular slot of variable backing distance and aperature length and width. The slot was assumed to be located immediately downstream of the injector. Tuned absorbers were designed for three different backing distances for both the first and third transverse modes of oscillation, and the decay rates for the chamber under consideration were then calculated. The effectiveness of each absorber design was also evaluated for three types of off design situations. The first of these was the case where the real part of the combustion response factor was larger than the design value. The second was the case where the real part of the combustion response factor was smaller than the design value. The third type of off design situation was for a chamber mode of oscillation different from the design mode.

Results of the study indicated a very strong dependence of the decay rate on the backing distance. It was also found that as either Mach Number or length to radius ratio for the chamber increased the combustors became relatively more unstable for a given backing distance. The calculations indicated that a first transverse mode design absorber was more successful at damping third transverse mode oscillations than was a third transverse mode design at damping first transverse mode oscillations,

and that, in fact, third transverse mode designs could drive first transverse mode oscillations. The absorbers were, in general, less effective at off design combustion response values, though with sufficiently large backing distances the damping effect of the absorbers was often large even in these cases.

Decay rates (log damping rates) were calculated using an integral equation iteration technique which calculates successively better approximations for the velocity potential and complex frequency. The analytical technique does not restrict the magnitude of the mean chamber Mach Number and is designed to accommodate the discontinuous boundary conditions which arise when a partial length acoustic absorber of the type used in this analysis is present. Convergence of the numerical technique used was sufficiently rapid to recommend the future use of this analytical technique in the evaluation of acoustic absorber performance in future or existing rocket combustion chambers.

INTRODUCTION

It is well known that acoustic absorbing devices can have a strong positive effect on the combustion stability of liquid rocket motors. It is therefore very desirable to have a method of evaluating analytically the potential effect of such acoustic absorbers on the stability of rocket engines.

Studies of the effects of full length acoustic liners on the stability of combustion chambers have been performed both for the simple case of a chamber with no mean flow, nozzle, or combustion zone (see Ref. 2) and for the more complex case of a chamber with finite mean flow, concentrated combustion, and short nozzle (see Ref. 1). More recently a method of evaluating the effects of partial length acoustic liners has been developed which also includes the effects of mean flow, a short nozzle and a concentrated combustion zone (see Ref. 4). This latter work is of the greatest practical interest since it is usual that acoustic absorbers line only a fraction of the chamber length in real engines.

The results of this last mentioned study were given in terms of stability curves for the chamber using a combustion response factor which characterized the combustion process. For design and evaluation of rocket engine stability, however, it is often more useful to know the damping rate produced by a given absorber in a given engine for a given combustion response. It is necessary to modify the method mentioned above in order to give results in this form, and to include a frequency sensitive absorber impedance. It is the purpose of this report to present the modified theory and to show the results of applying it to seven rocket engine designs.

The type of absorber considered in this work was a simple circumferential slot of variable backing distance, aperture length, and aperture width. (See Fig. 1). The seven engines evaluated provided a rather large cross section of chamber design parameters such as mean Mach Number, length to radius ratio and ratio of specific heats. The influence of these parameters on the stability of chambers with acoustic absorbers of this type could therefore be determined.

Decay rates are calculated using a modification of the method of analysis mentioned above and in Ref. 4. This modified analysis is an integral equation iteration technique in which the iterations are carried out numerically on a digital computer. The oscillations are assumed to be linear (small amplitude), and to occur in a homentropic and irrotational source free field of calorically perfect gases. The combustion processes are represented using the sensitive time lag model.

Results are given in terms of log damping rates for a given engine-absorber combination and combustion response (location on the neutral stability curve for the unlined chamber). Curves for only the first and third transverse primary modes of oscillations are presented here, although some calculations have also been performed for the second and fifth transverse modes.

THEORY

Combustor Model

Combustion chambers of cylindrical cross section terminated by a "short" nozzle and having all combustion concentrated at the injector face are the type considered in this report. The combustion zone mass generation response is assumed to be pressure dependent only and can be represented using the sensitive time lag model devised by Crocco (3). An acoustic absorber is assumed to be located immediately downstream of the injector. This absorber is basically a radially oriented circumferential slot. Its geometry and location are shown in Figure 1. The rest of the cylindrical periphery of the chamber is taken to be non-absorbing. The gasdynamic field downstream of the combustion zone is assumed to be composed of a single component, calorically perfect combustion product gas. The flow is taken to be homentropic and irrotational.

Using these assumptions the conservation equations for mass and momentum can be written down. (See for example Reference 1). These equations are then linearized using the small perturbation assumption. Thus $p = 1 + p'$, $\rho = 1 + \rho'$, $a = 1 + a'$, $u = M + u'$, $v = v'$, $w = w'$ where all state variables are non-dimensionalized by their mean values, velocities are made non-dimensional by division with the mean sound speed and primes indicate perturbation quantities. The perturbed variables are represented as the product of a space dependent function and $e^{i\omega t}$ where ω is the complex frequency ($\omega = Re(\omega) + i\lambda$) and t is time. Frequency is non-dimensionalized by multiplication with the ratio of the chamber radius to the mean sound speed, time is non-dimensionalized by division with the

same ratio, and space variables by division with the chamber radius.

Because of the irrotationality assumption it is possible to reduce the perturbed conservation equations to a single partial differential equation in ϕ, the space dependent part of the perturbation velocity potential. This equation and the expressions relating ϕ to the other perturbed dependent variables are given in Appendix A.

Introduction of a Green's function for the chamber and the use of Green's Theorem make possible the transformation of this partial differential equation and its associated boundary conditions into the two governing integrals equations which follow.

$$\phi = \Omega_N + \iiint_V G_N(\vec{r}/\vec{r}_o)\left(M^2\frac{\partial^2\phi}{\partial z^2} + 2Mi\omega\frac{\partial\phi}{\partial z}\right) dV_o + \iint_S G_N(\vec{r}/\vec{r}_o)\beta\left(\gamma i\omega\phi + \gamma M\frac{\partial\phi}{\partial z}\right) dS_o \qquad \text{.........} \quad (1)$$

$$\omega^2 - \eta_N^2 = \iiint_V \Omega_N\left(M^2\frac{\partial^2\phi}{\partial z^2} + 2Mi\omega\frac{\partial\phi}{\partial z}\right) dV + \iint_S \Omega_N\beta\left(\gamma i\omega\phi + M\frac{\partial\phi}{\partial z}\right) dS \qquad \text{.........} \quad (2)$$

In the equations above the volume integrals are taken over the complete chamber volume and the surface integrals over all the boundary surfaces of the cavity (cylindrical walls, injector face and nozzle exit plane). The quantities β, Ω_N, η_N, M and G_N which appear in the equations have the following meanings:

β — specific acoustic admittance over which the integral is being taken. For the nozzle exit plane $\beta_N = M(\frac{\gamma-1}{2\gamma})$. For the cylindrical solid walls $\beta_C = 0$. For the absorber slot width, W_A, $\beta_C = \frac{1}{\gamma K}$, where K is the

specific absorber impedance and is a function only of $Re(\omega)$ when the absorber geometry is fixed. For the injector plane (combustion zone)

$$\beta_i = M[n(i-e^{-i\omega\tau}) - \frac{1}{\gamma}].$$

M mean chamber Mach Number.

Ω_N one of the acoustic eigenfunctions for the chamber with no mean flow or liner.

η_N the eigenvalue associated with the eigenfunction Ω_N.

$G_N(\vec{r}/\vec{r}_o)$ the Green's function for the chamber. (See Appendix A for exact definition)

Solution of the Equations

An iterative technique is used to solve the integral equations for ω and ϕ. First an initial functional form for ϕ and an initial value for ω are substituted into the integrals on the right hand side of the equations. The initial values used in this work were the velocity potential and frequency for the chamber with no liner. These quantities can be obtained using a separation of variables technique. Integration using these initial values then produces first approximations for ω and ϕ . These quantities are in turn substituted into the integrals to yield second order approximations. This process continues until ω and ϕ are suitably invariant between successive approximations. Typically ten iterations yield results that vary less than one tenth of one percent. A more detailed description of the iteration process used is given in Appendix A. A description and print out of the computer program used in the iterative calculations is given in Appendix C.

RESULTS AND DISCUSSION

Seven different engine designs were considered. Table 1 gives the necessary geometrical and mean operating parameters for these combustors. For each of the combustion chambers tuned absorbers (i.e. Im(K) = 0) were designed for two or more primary transverse modes of oscillation. All the absorbers were tuned for a combustion response value corresponding to the minimum point on the n,τ neutral stability curve for the chamber without an absorber. An example of this point is shown in Figure (2) for Engine 1 as point A on the curve.

The acoustic absorbers evaluated in this work were made to satisfy one of two design conditions. The majority of the absorbers were designed subject to the condition that the absorber aperture length be equal to the backing distance. Some of the absorbers, however, were designed in such a way as to maximize an "effective decay coefficient" for a given backing distance. This latter requirement in general yielded aperture lengths less than the backing distance. The "effective decay coefficient" was introduced in order to provide an approximate means of measuring the broad frequency band effectiveness of an absorber. Its exact definition and a discussion of its role in determining absorber designs in this work are presented in Appendix B.

Once either one of these absorber design conditions was adopted for a particular engine design, absorber backing distance, and mode of oscillation, it was possible to calculate chamber frequencies and decay rates and absorber slot widths using the two governing integral equations, an expression for the impedance of the absorber (See Appendix B), and

the condition that the liner be tuned (Im(K) = 0). Tuned absorbers of this type were designed for several backing distances for a given chamber.

If an absorber is designed so that it is tuned for a given chamber, a particular value of the combustion response, and a given primary mode of oscillation, then when either the combustion response value or the mode of oscillation is changed the absorber will not be tuned for the new conditions. For each absorber design studied at least two off design combustion response values and one off design mode were investigated in order to determine the general effectiveness of the absorber away from its tuning point.

Tuned Absorber Results

Curves of decay coefficient (imaginary part of the complex frequency) as a function of backing distance to chamber radius ratio for tuned first transverse mode absorbers for the seven chambers considered are shown in Figure (3). The curves for each of the seven combustors are seen to approximate straight lines on the log-log plot. The slope of the lines is approximately three, indicating that the decay coefficient increases as the cube of the backing distance. All the absorbers used in Figure (3) were designed subject to the condition $L_a = B$. The straight lines for the different engines, though approximately parallel, are in some cases displaced from each other a considerable distance. This means that the value of the decay coefficient for a given backing distance will be much lower in some engines than in others. For example, comparison of the lines representing Engines 3 and 4 shows that at a B/R value of 0.10 the

decay coefficient for Engine 3 is about 290 sec^{-1} while for Engine 4 it is only about 14 sec^{-1}. The difference in liner effectiveness is related to chamber design parameters such as Mach Number and length to radius ratio as will be shown shortly.

The dependence of the decay coefficient on absorber slot width for the same engines, absorbers and mode as in Figure (3) is shown in Figure (4). The slot widths shown for any engine are those that tune an absorber in that engine for the backing distances of Figure (3). The absorber geometry in any engine for a particular decay coefficient is found using Figures (3) and (4) by simply picking the decay coefficient desired, noting the point of intersection of the line of constant decay coefficient with the desired engine line, and then reading off the corresponding values of B/R and W_a/R.

An attempt was made to correlate all the tuned absorber decay coefficient results for first transverse mode on a single plot. Figure (5) displays the results of this correlation when decay coefficient is plotted against the parameter

$$\alpha = \left(\frac{B}{R}\right)^3 \left(\frac{L}{R}\right)^{-1} \frac{1}{M^{1/3}} \left(\frac{a_o^*}{R^*}\right)$$

It is seen that all the absorbers evaluated fall fairly close to a line of slope approximately one. Since the plot is on a log-log scale, the scatter of the design points makes it clear that the attempted correlation is only roughly successful. It does, however, provide a means of obtaining an estimate of the backing distance required in order to obtain a given decay coefficient. It also indicates the effects of length to

radius ratio and Mach Number on the decay coefficient. Increasing either of these parameters decreases the decay coefficient for a fixed backing distance. The dependence is linear in the case of length to radius ratio and a 1/3 power dependence in the case of Mach Number. Because of the wide range of Mach Numbers and length to radius ratios examined, it is felt that some confidence can be placed in these dependencies. It should be remembered that the correlation of Figure (5) is valid strictly for tuned first transverse mode absorbers with $L_a = B$.

Third transverse mode tuned absorbers with three or more different backing distances were only designed for three of the engines studied, though at least one third transverse mode absorber was designed for all seven engines. Decay coefficients as functions of backing distance for the three engines evaluated most extensively are shown in Figure (6). The three straight lines in the figure are those that result when the condition $L_a = B$ is used in the absorber design. The dashed lines in the figure represent absorbers designed subject to the maximum effective decay coefficient condition discussed in Appendix B. Until B/R was about 0.09 it was impossible to satisfy this design requirement. (That is, the optimum effective decay coefficient occurred at L_a values greater than B). Thereafter absorbers designed in such a way as to maximize the effective decay coefficient produced decay rates considerably smaller than those designed so that $L_a = B$. An apparent trade off exists between these two types of absorbers at these larger backing distances in that an absorber designed for optimum broad band effectiveness produces a smaller decay coefficient under tuned conditions than does an absorber designed so that

$L_a = B$. Comparison of Figure (6) with Figure (3) shows that, for the same backing distance, absorbers designed so that $L_a = B$ produce substantially larger decay coefficients for third transverse mode oscillations than they do for the first transverse mode. Figure (7) exhibits the dependence of the decay coefficient on the slot width for tuned third transverse modes. It can be used in conjunction with Figure (6) to determine absorber design dimensions, as was the case for first transverse mode absorbers.

Off Design Results - Combustion Response

The effectiveness of an absorber in damping oscillations occurring at combustion response (n,τ) values different from design values was evaluated by determining decay coefficients for two points on the neutral stability curve for the given combustor with no absorber. One of these points was at a frequency higher than the design (minimum n) frequency, the other was at a frequency lower than the design frequency. Examples of these two points are shown in Figure (2) for the first transverse mode in Engine 1 as points B and C, respectively. Point A on the curve is the design point. Values of ω/ω_o, the real part of the frequency divided by the acoustic frequency for the first transverse mode, are shown as parametric points on the neutral stability curve. The effectiveness of the liner at the off design points varied between different chambers, different modes and different backing distances. For the example chosen, Engine 1, first transverse mode, backing distance B = 0.10, the decay coefficient at point A, the design point, was 123 sec^{-1}. At point B,

the high frequency point, it was -40 sec^{-1}. At point C, the low frequency point, it was 178 sec^{-1}. The neutral stability curves for this particular chamber with and without absorber are shown in Figure (8). Two neutral stability curves are shown. The solid curve is for the chamber with no absorber; the dashed curve is the neutral stability limit for the chamber with the absorber. Areas inside neutral stability curves are linearly unstable (growth occurs); areas outside are linearly stable (decay occurs). Since the dashed curve is predominantly inside the solid curve, the absorber provides an overall stabilizing effect for most of the frequency range of the first transverse mode. It is seen, however, that at frequencies above ω/ω_o = 1.04, the solid curve lies inside the dashed curve. For this region the addition of the liner is destabilizing. For example, point B, which is in this region, has a decay coefficient of -40^{-1} which indicates linear growth rather than decay. Without the absorber oscillations at point B neither grow nor decay. The general trends in the results of off design calculations of this type indicate that for frequencies above the design frequency the absorber is usually less effective and produces smaller decay coefficients than at design. For very high frequencies close to the transition from a pure transverse mode to a combined transverse longitudinal mode it is possible for the absorber to be driving rather than damping, as in the example discussed above. For off design frequencies below the design frequency the absorber usually produced decay rates of the same order as those at design; in some cases larger than at design. Overall it was concluded that the stability improving effect of the absorber is quite dependent upon the value of the combustion response

and its location relative to the design value. In order to guarantee stability improvement for a given chamber, absorber, and mode combination, the entire range of combustion response factors of interest must be evaluated.

Off Design Results - Higher Modes

For all the first transverse mode absorbers designed, calculations were performed to determine the decay coefficients produced by them for the third transverse mode of oscillation. Some second transverse mode calculations of this type were also made. Typically the results of such calculations indicated that first transverse mode absorbers are quite effective in damping higher mode oscillations. Figure (9) shows the decay coefficients produced when a first transverse mode absorber was used to damp third transverse oscillations in three of the engines studied. Comparison with Figure (3) shows that for small backing distances larger decay coefficients are produced for the third transverse mode than for the first transverse mode even though the absorber is tuned for first transverse mode oscillations. As backing distances become larger, however, the fact that the slopes of the tuned first transverse mode lines (Figure (3)) are considerably larger than the slopes in Figure (9) ensures that their damping coefficients will eventually be larger. This effectiveness of the first transverse mode absorbers in damping third transverse mode absorbers is not too surprising in light of the fact that for the same backing distance tuned third transverse absorbers provide decay coefficients for the third transverse mode about an order of magnitude larger than do tuned first transverse mode absorbers for the first

transverse mode (compare Figures (3) and (6)).

Off Design Results - Lower Modes

Calculations were also performed in order to determine how effective higher mode absorber designs were at damping lower mode oscillations. Results indicated not only that using higher mode designs for lower mode oscillations always produced rather small damping coefficients, but also that in many cases these coefficients actually were negative. That is, the absorber drove the lower mode oscillations. This type of situation is demonstrated in Figure (10). Figure (10a) shows decay coefficients as functions of B/R for the first transverse mode of oscillation with absorbers tuned for third transverse oscillations for three different engines. It can be seen that for backing distances less than about 0.15 the decay coefficients are negative. As the backing distance increases beyond 0.15 all three engines eventually show some damping effect with the third transverse absorbers. The stability condition of Engine 2 with an absorber of backing distance 0.10 is shown in Figure (10b) using neutral stability limits. The dashed curve is a neutral stability limit for the first transverse mode using the absorber designed for the third transverse mode. The solid curve is the neutral stability limit for the engine with no absorber at all. It is seen that for the entire first transverse mode the solid curve is inside the dashed curve. This means that the third transverse mode absorber is destabilizing with respect to the first transverse mode for any value of combustion response.

A higher mode absorber used to damp lower mode oscillations always

produced a negative imaginary part for the acoustic impedance of the absorber. Using a lower mode absorber to damp higher mode oscillations always produced a positive imaginary part for the acoustic impedance. The dependence of the non-dimensional decay coefficient on the imaginary part of the absorber impedance for a fixed value of the real part of the impedance is shown in Figure (10c) for the first transverse mode of oscillation in Engine 2. It is seen that maximum damping occurs at tuning, that is, where $Im(K) = 0$. Increasing the imaginary part of K decreases λ until as $\lambda \to \infty$ no damping effect is produced. Decreasing the imaginary part of K ($Im(k) < 0$) produces a driving effect that first increases and then decreases to zero. It is to be noted that an extreme case has been presented; if $Re(k)$ were smaller (more acoustic resistance) the entire curve in Figure (10c) would be shifted upwards and λ would not necessarily ever be negative. The behavior of λ, however, would always be asymmetrical with respect to $Im(K) = 0$ with values of $Im(K) < 0$ providing less damping than values of $Im(K) > 0$. It is important to note that this type of asymmetrical behavior is not reflected using the absorption coefficient. As it is usually defined ($\alpha = 4Re(K)/([Re(K) + 1]^2 + [Im(K)]^2)$ the absorption coefficient is absolutely symmetrical with respect to $Im(K) = 0$. The conclusion reached here is that the value of the absorption coefficient alone is not sufficient to determine the effectiveness of an absorber. For the same value of α the damping of a given absorber can be quite different depending on the sign of the imaginary part of the acoustic impedance.

SUMMARY OF CONCLUSIONS

An analytical method for predicting decay coefficients in rocket combustors with partial length acoustic absorbers has been developed. Application of this method to seven different combustors for a particular type of absorber design led to the following conclusions.

1. The technique was applicable and practicable for all combinations of absorbers, acoustic modes and engines tested.
2. It was possible to correlate all the decay coefficients for the first transverse mode tuned absorbers using a correlation coefficient which involved the chamber length to radius ratio and mean Mach Number. From this correlation it was concluded that decay coefficients increased with the cube of the absorber backing distance and decreased with Mach Number to the 1/3 power and length to radius ratio to the first power.
3. For values of the combustion response other than that value for which the absorber was tuned, the absorber was often less effective. In fact for some values of combustion response the absorber provided a driving rather than damping effect.
4. When an absorber tuned for a particular mode was used to damp a higher acoustic mode it was quite effective.
5. When an absorber tuned for a given acoustic mode was used to damp a lower mode it was generally very ineffective and in many cases provided a driving rather than damping effect.

TABLE I

Engine Number	Mach Number	Ratio of Specific Heats	Length to Radius Ratio	Radius of Chamber	Speed of Sound
1	0.265865	1.146	1.984159	9.07 in.	5181.7ft/sec.
2	0.304389	1.148	1.841964	8.47 in.	5248.1
3	0.392677	1.149	1.051433	7.59 in.	5281.6
4	0.154893	1.343	5.440007	5.03 in.	5569.6
5	0.081879	1.3387	3.732183	4.44 in.	5620.2
6	0.064638	1.3835	5.662040	3.17 in.	5512.3
7	0.191492	1.348	6.796209	1.26 in.	5740.6

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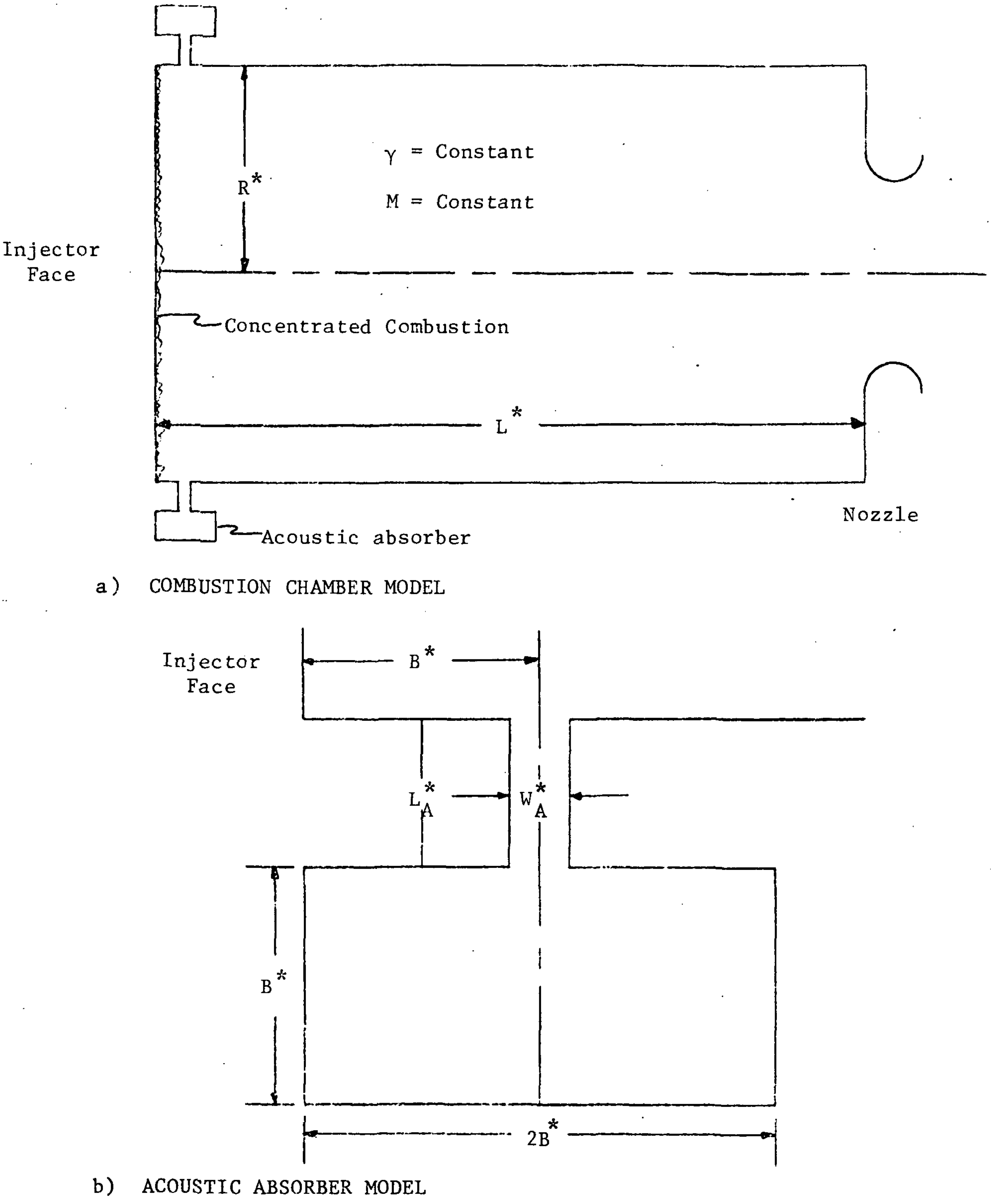


Figure 1 Chamber and Absorber Configurations

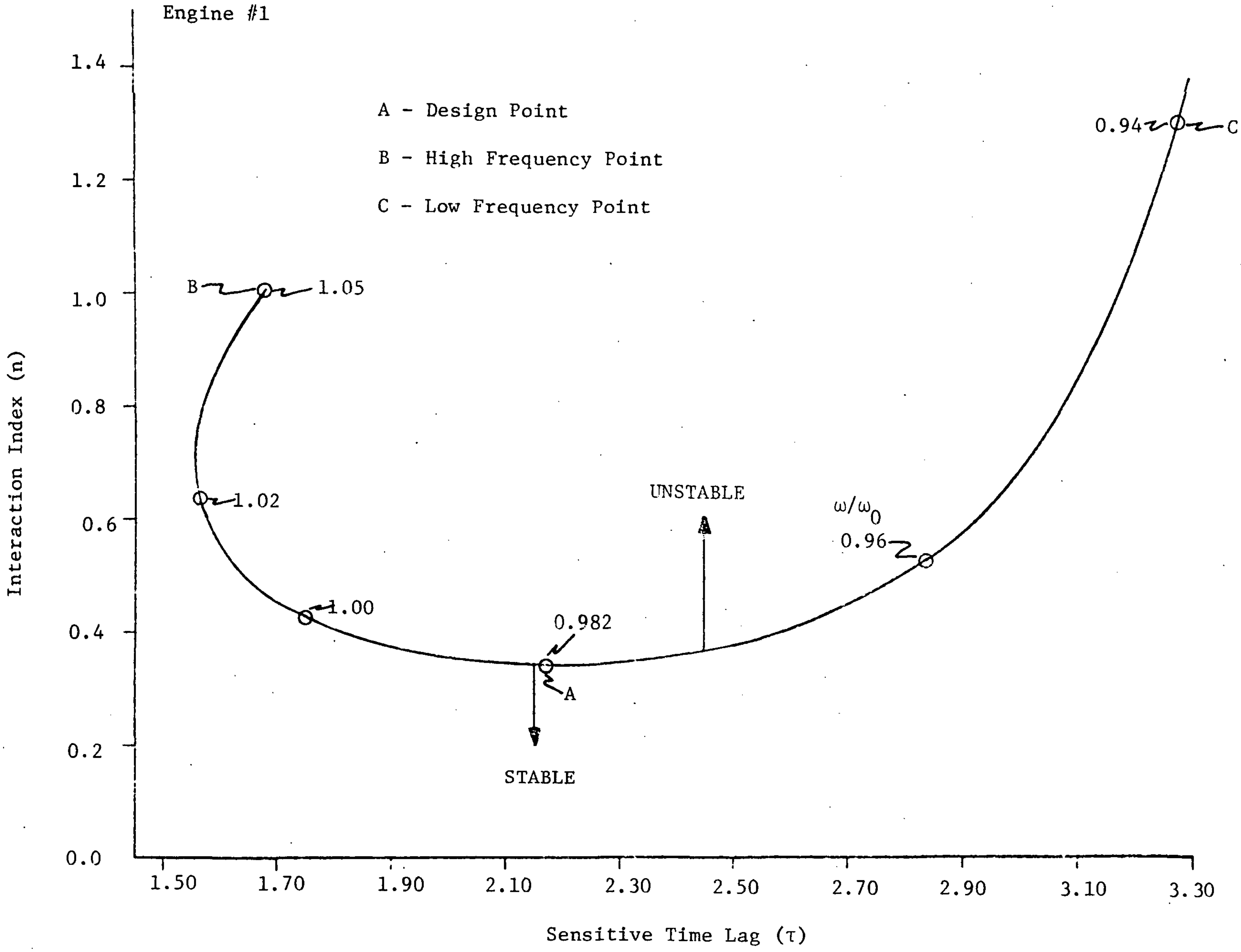


Figure 2 Neutral Stability Curve for Chamber with No Absorber Showing Combustion Response Design Points

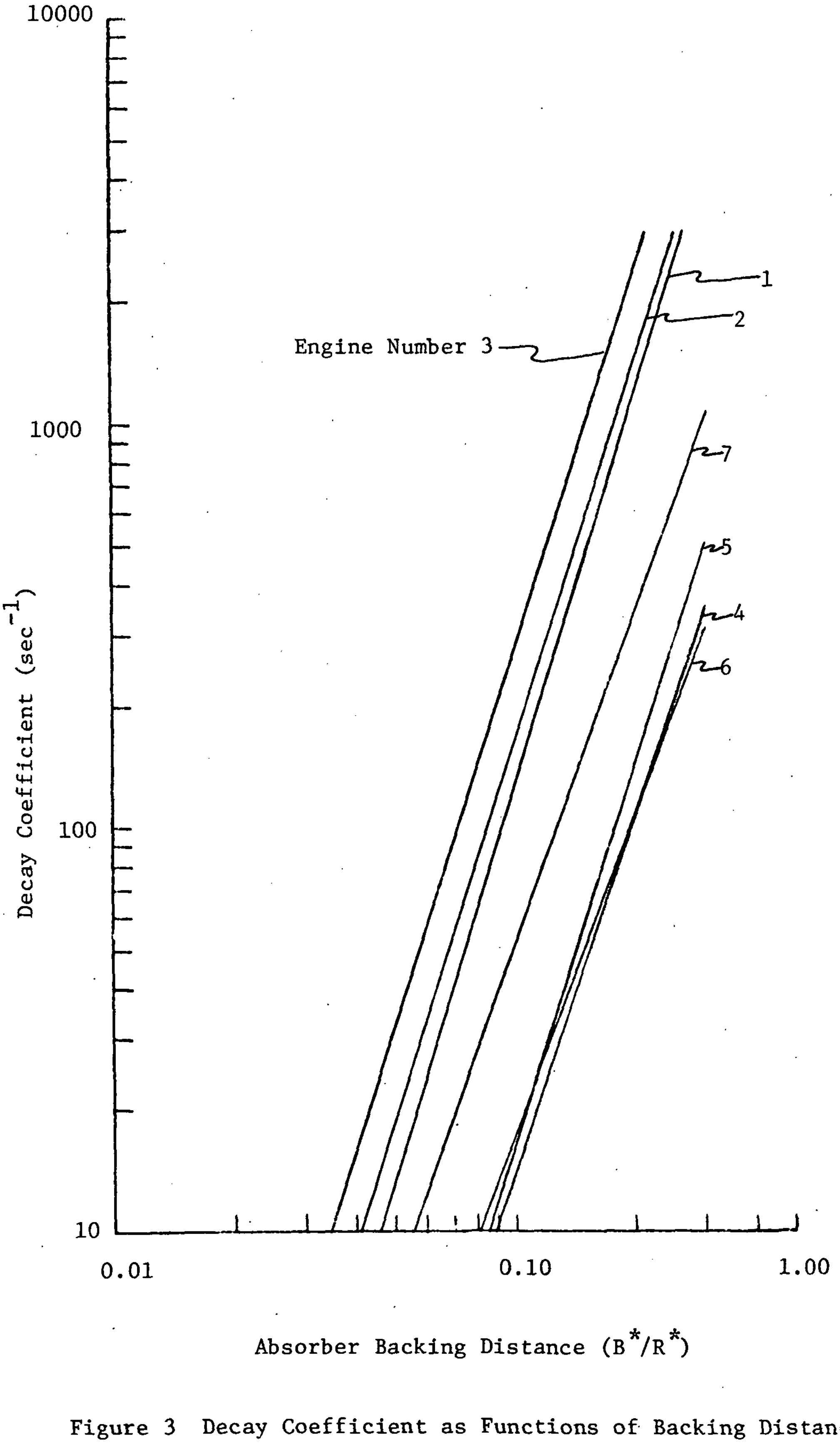


Figure 3 Decay Coefficient as Functions of Backing Distance for First Transverse Mode Absorbers

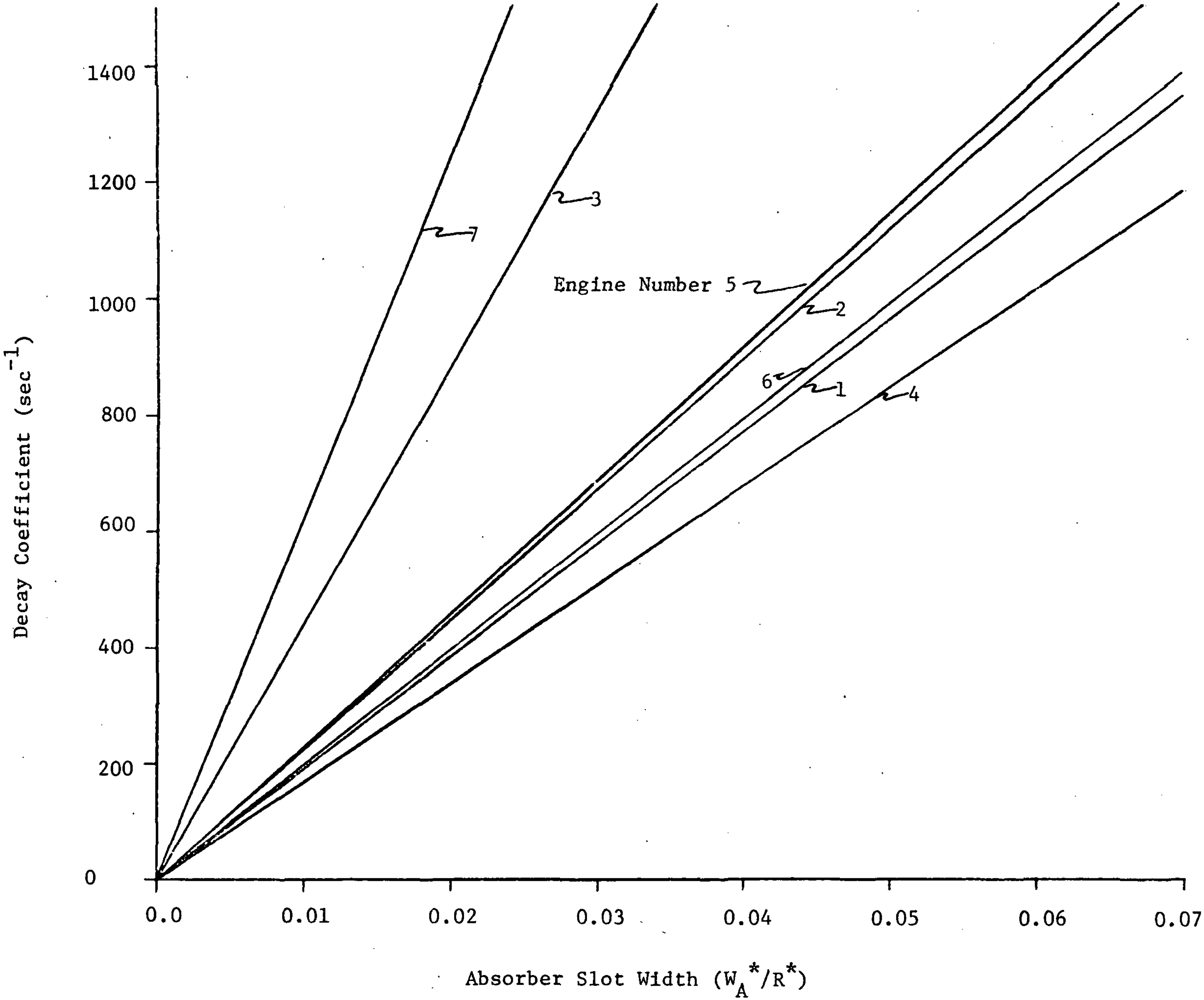
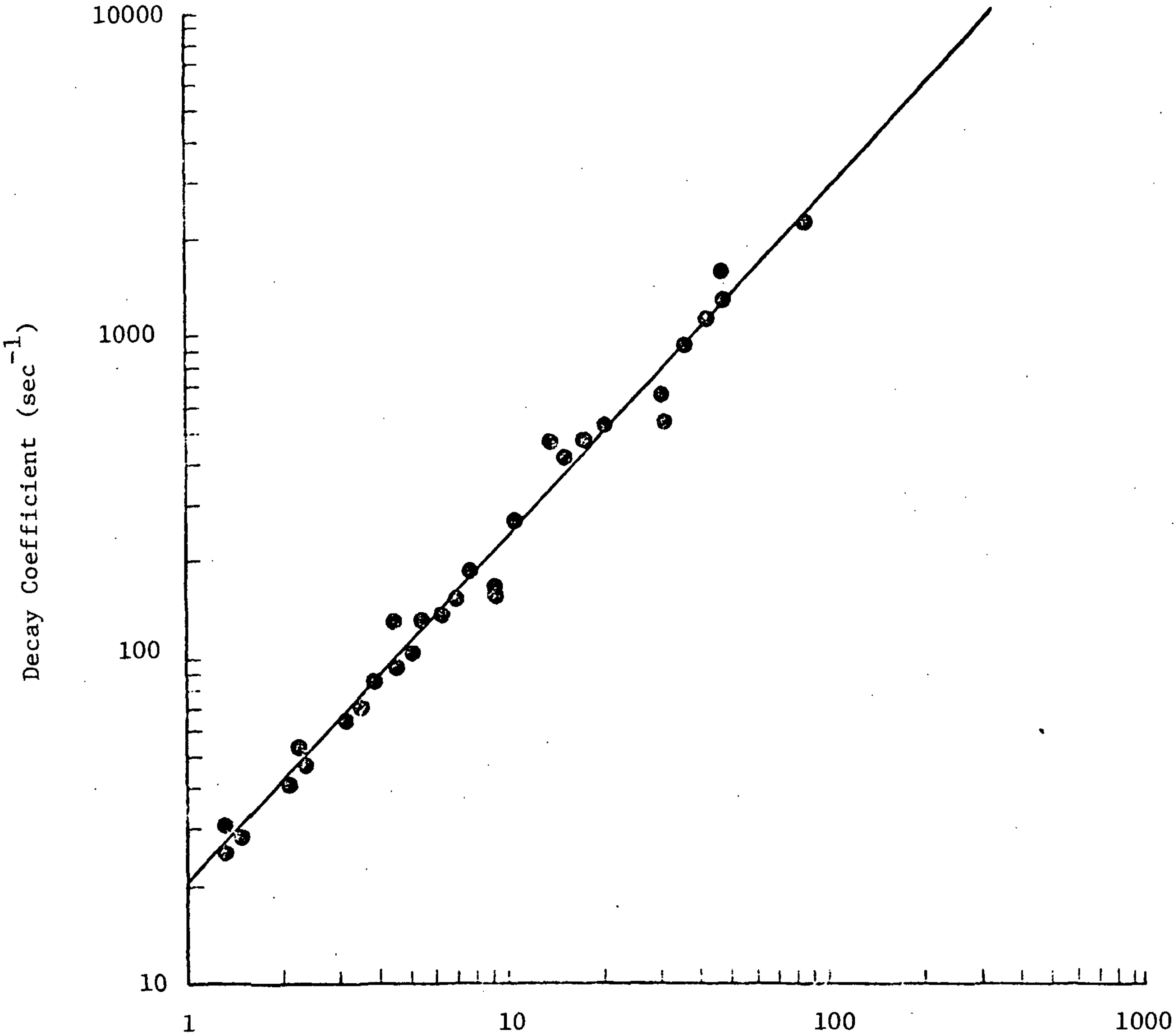


Figure 4 Decay Coefficient as a Function of Slot Width for First Transverse Mode Absorbers



$$\alpha = \left(\frac{B^*}{R^*}\right)^3 \left(\frac{R^*}{L^*}\right)\left(\frac{1}{M^{1/3}}\right)\left(\frac{a_o^*}{R^*}\right) \quad (\mathrm{sec}^{-1})$$

Figure 5 Correlation of First Transverse Decay Coefficient Results

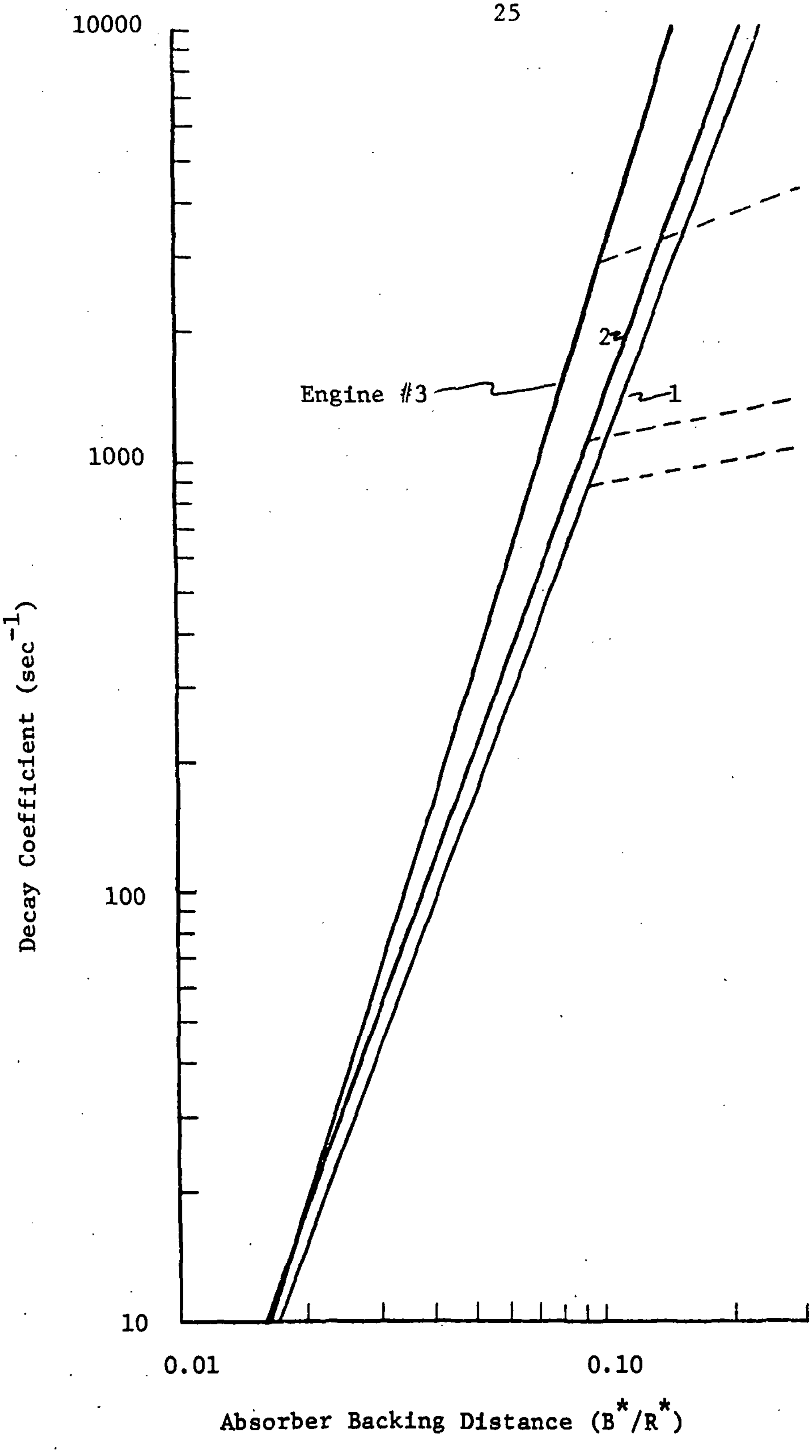


Figure 6 Decay Coefficient as a Function of Backing Distance for Third Transverse Mode Absorbers

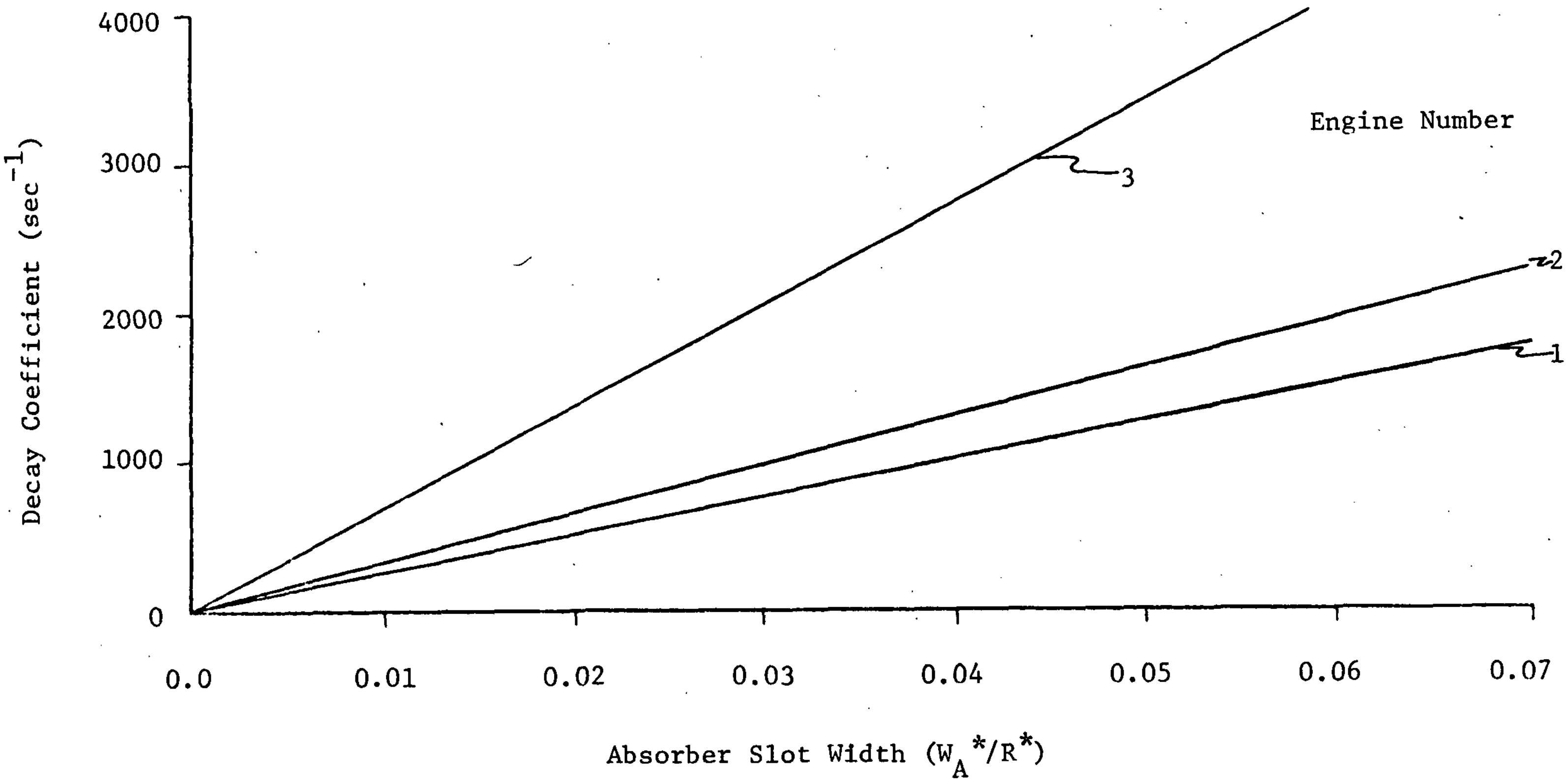


Figure 7 Decay Coefficient as a Function of Slot Width for Third Transverse Mode Absorbers

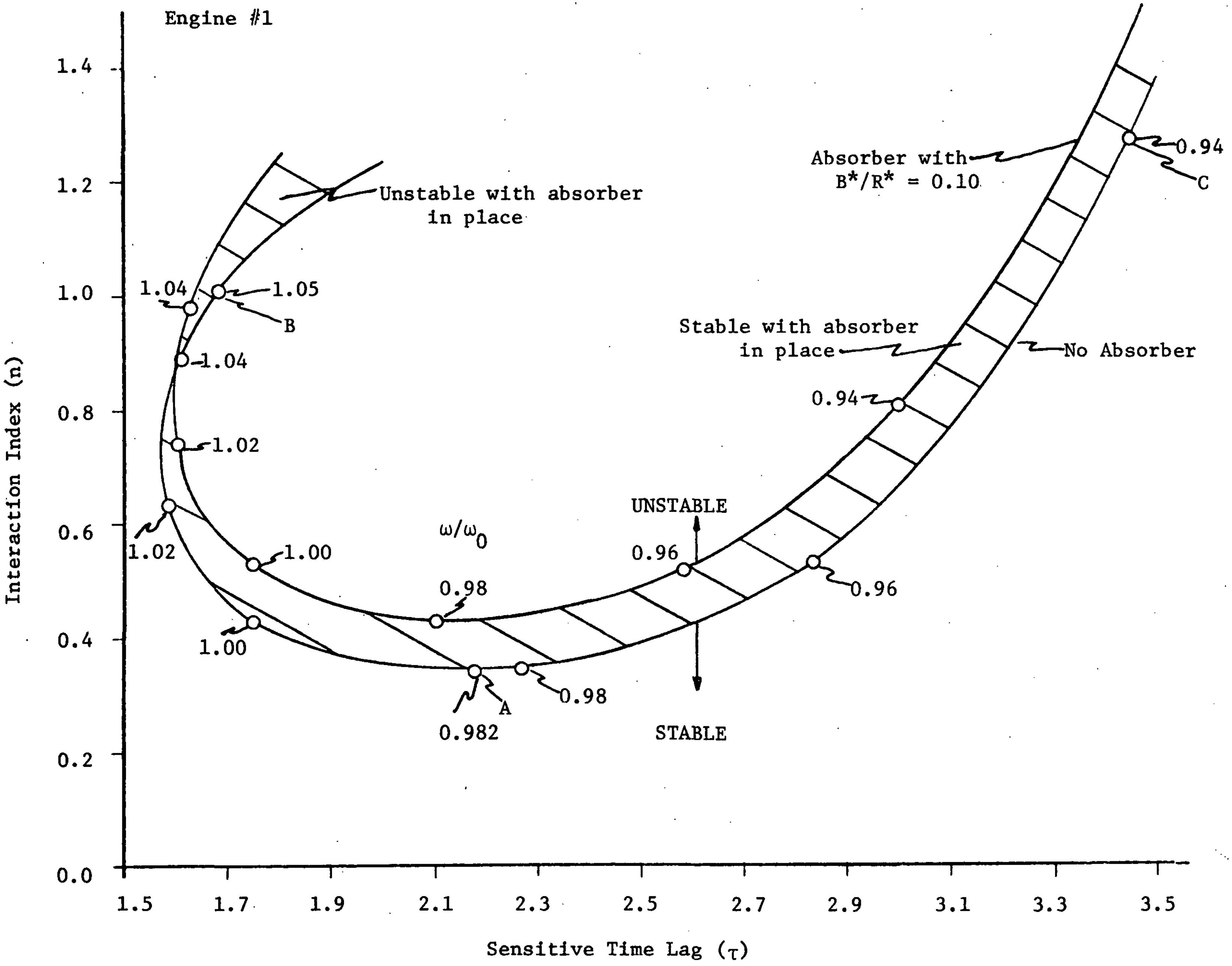


Figure 8 Neutral Stability Curves Showing the Effect of an Absorber

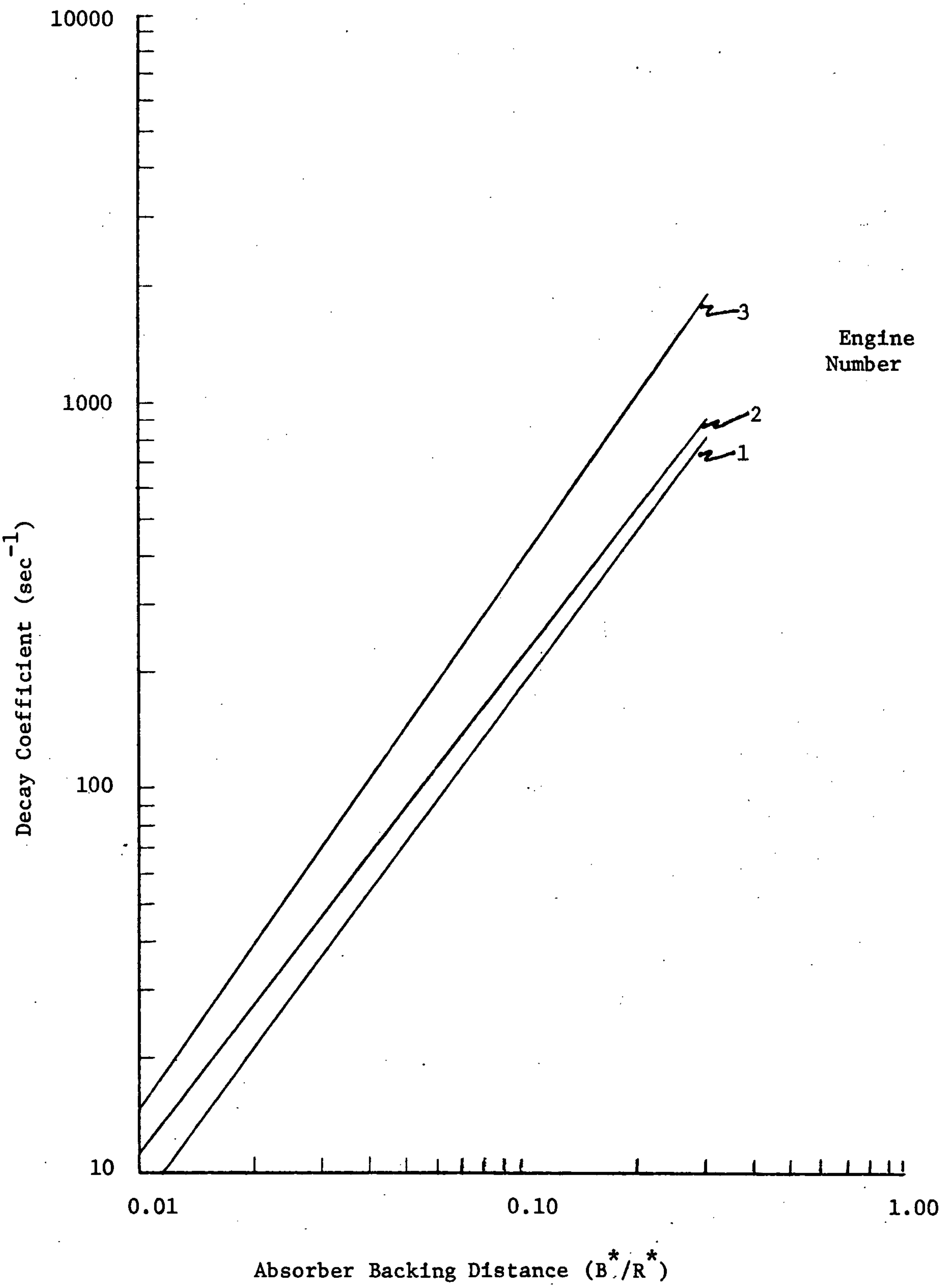


Figure 9 Decay Coefficients for First Transverse Mode Design Absorbers Used to Damp Third Transverse Mode Oscillations

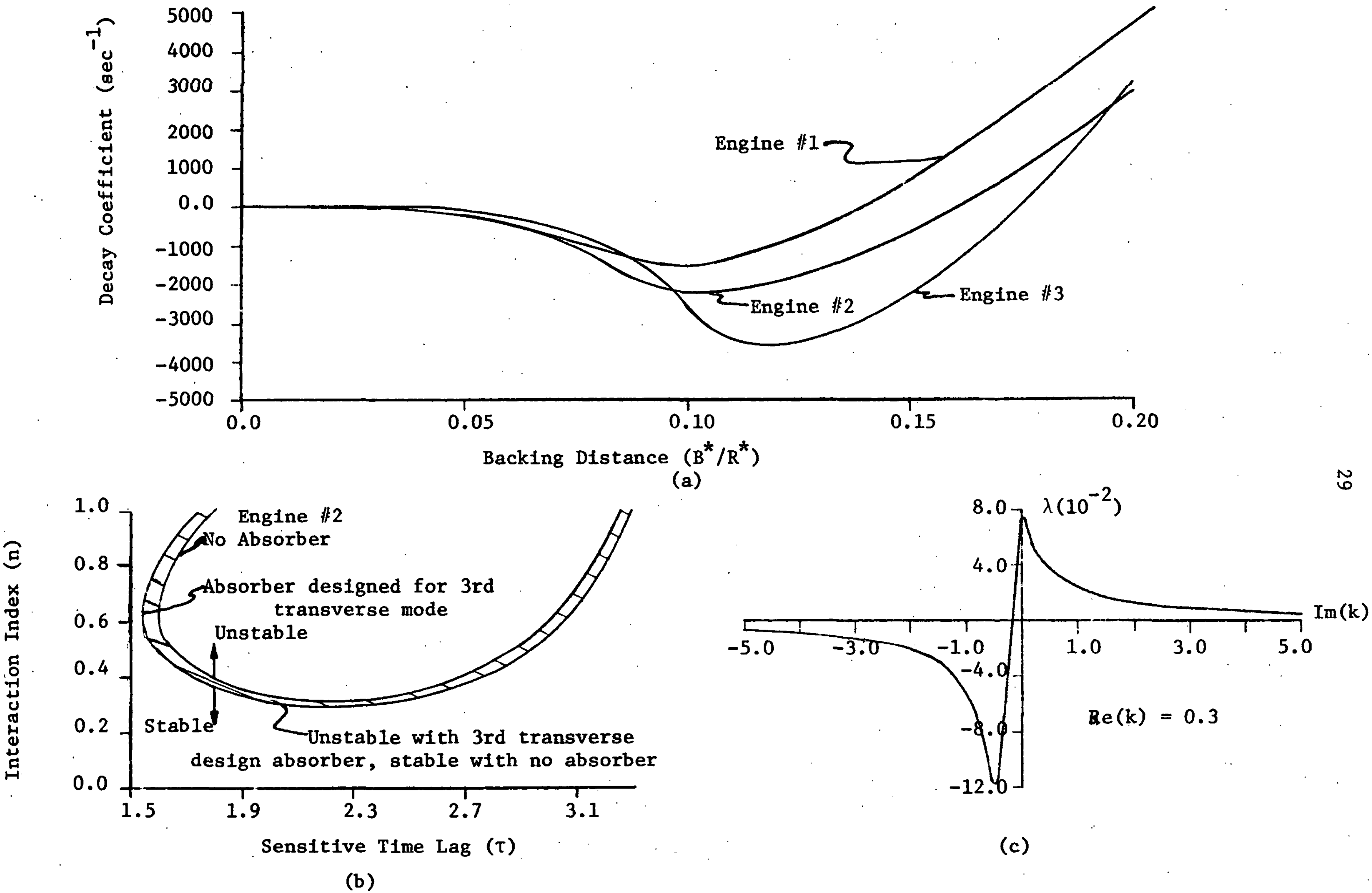


Figure 10 Third Transverse Design Used to Damp First Transverse Mode

APPENDIX A

Basic Equations

The perturbation equation describing the oscillatory flow in the combustion chamber written in terms of the perturbation velocity potential is

$$\nabla^2\phi + \omega^2\phi = M^2\frac{\partial^2\phi}{\partial z^2} + 2Mi\omega\,\frac{\partial\phi}{\partial z} \tag{A-1}$$

This equation is derived from the conservation equations for the chamber in Reference (1). Perturbation pressure and velocity are related to ϕ through the following relationships

$$p' = -\gamma\left(i\omega\phi + M\,\frac{\partial\phi}{\partial z}\right)$$

$$\vec{q}\,' = \vec{\nabla}\vec{\phi}$$

Equation (A-1) can be transformed into the two integral equations Equations (1) and (2) through the introduction of the Green's Function satisfying the following equation

$$\nabla^2 G + \omega^2 G = \delta(\vec{r}-\vec{r}_o) \tag{A-2}$$

where $\vec{\nabla}G \cdot \vec{n} = 0$ on all boundaries and δ is the delta function. The Green's function is represented as an expansion in tha orthonormal eigenfunctions for the chamber with no combustion or mean flow and solid boundaries as

$$G = \sum_{\ell}\sum_{m}\sum_{n}\frac{\Omega_{\ell mn}(\vec{r})\;\Omega_{\ell mn}(\vec{r}_o)}{\omega^2 - \eta^2_{\ell mn}}$$

where $\Omega_{\ell mn}$ is one of the orthonormal eigenfunctions given by

$$\Omega_{\ell mn} = \cos m\theta \; J_m(\lambda_{\ell m} r) \cos \frac{n\pi z}{L} \cdot \frac{1}{\Lambda^{\frac{1}{2}}_{\ell mn}}$$

and
$$\eta^2_{\ell mn} = \frac{n^2\pi^2}{L^2} + \lambda^2_{\ell m} \quad .$$

The quantity $\lambda_{\ell m}$ is a root of the equation $\left[J'_m(\lambda_{\ell m} r)\right]_{r=1} = 0$ and $\Lambda_{\ell mn}$ is determined by the condition $\iiint_V \Omega_{\ell mn}\,\Omega_{\ell mn} dV = 1$

The modified Green's function G_N which appears in Equations (1) and (2) is just the Green's function with one term in the series removed. That is, the term with $\ell = \hat{\ell}$, $m = \hat{m}$, $n = \hat{n}$ where $\hat{\ell}$, $\hat{m}$, $\hat{n}$ are arbitrary, is deleted from the series for G_N. Thus

$$G_N = \sum_{\ell}\sum_{m}\sum_{n} \frac{\Omega_{\ell mn}(\vec{r})\;\Omega_{\ell mn}(\vec{r}_o)}{\omega^2 - \eta^2_{\ell mn}} \left(1 - \delta(\ell,\hat{\ell})\delta(m,\hat{m})\delta(n,\hat{n})\right)$$

where $\delta(i,j) = 1$ if $i = j$ and $\delta(i,j) = 0$ if $i \neq j$.

In Equation (1) the quantity Ω_N can be interpreted as $\Omega_{\hat{\ell}\hat{m}\hat{n}}$ and in Equation (2) η_N is equivalent to $\eta_{\hat{\ell}\hat{m}\hat{n}}$.

In the solution of Equations (1) and (2) the velocity potential and frequency for the chamber with no liner but with mean flow and combustion are used as first approximations to ϕ and ω. These quantities are called respectively $\tilde{\phi}$ and $\tilde{\omega}$. The form of $\tilde{\phi}$ can be found using the technique of separation of variables and is given by

$$\tilde{\phi} = \frac{\cos \hat{m}\theta}{\Lambda^{\frac{1}{2}}_{\hat{\ell}\hat{m}\hat{n}}\;\psi} J_{\hat{m}}(\lambda_{\hat{\ell}\hat{m}} r)\left(e^{izB_1} + C\, e^{izB_2}\right) \tag{A-3}$$

where

$$B_1 = \frac{\hat{\omega}M + \left[\tilde{\omega}^2 M^2 + (M^2-1)(\lambda^2_{\hat{\ell}\hat{m}} - \tilde{\omega}^2)\right]^{\frac{1}{2}}}{(1 - M^2)}$$

$$+\int_{S_c}\beta_c G_N(\omega^{(j-1)})F_2(\tilde{\phi},\omega^{(j-1)})\,dS + \beta\int_{S-S_c} G_N(\omega^{(j-1)})F_2\ (\phi^{(j-1)}-\tilde{\phi})\omega^{(j-1)}\,dS$$

$$+\int_{S_c}\beta_c G_N(\omega^{(j-1)})F_2[(\phi^{(j-1)}-\tilde{\phi}),\omega^{(j-1)}]\,dS$$

$$+M\int_V G_N(\omega^{(j-1)})F_1[(\phi^{(j-1)}-\tilde{\phi}),\tilde{\omega}^{(j-1)}]\,dV \qquad \text{(A-4)}$$

$$\omega^{2\,(j)} = \omega^{2(1)} + i(\omega^{(j-1)}-\tilde{\omega})\left[\gamma\int_S \beta\Omega_N\tilde{\phi}\,dS + 2M\int_V \Omega_N\frac{\partial\tilde{\phi}}{\partial z}\,dV\right]$$

$$+\int_S \beta\Omega_N F_2\left[(\phi^{(j-1)}\quad\tilde{\phi}),\omega^{(j-1)}\right]d \quad + M\int_V \Omega_N F_1\left[(\phi^{(j-1)}\quad\tilde{\phi}),\omega^{(j-1)}\right]dV$$

where $F_2(\phi,\omega) = \gamma(i\omega\phi + M\frac{\partial\phi}{\partial z})$

$$F_1(\phi,\omega) = 2i\omega\frac{\partial\phi}{\partial z} + M\frac{\partial^2\phi}{\partial z^2}$$

$$H_N(\omega^{(j-1)},\tilde{\omega}) = \sum_{\ell}\sum_{m}\sum_{n}\frac{\Omega_{\ell mn}(\vec{r})\quad \underset{mn}{(\vec{r}_o)}}{(\omega^{2\,(j-1)}-\eta_N^2)(\tilde{\omega}^2-\eta_N^2)}(1-\delta(\ell,\hat{\ell})\delta(m,\hat{m})\delta n,\hat{n})$$

$S-S_c$ is the surface of the two ends of the chamber. S_c is the surface of the cylindrical walls of the chamber. The expressions above are valid for $j \geq 2$.

For $j = 1$

$$\phi^{(1)} = \tilde{\phi} + \int_{S_c}\beta\ G_N(\tilde{\omega})F_2(\tilde{\phi},\tilde{\omega})\,dS$$

$$\omega^{2\,(1)} = \tilde{\omega}^2 + \int_{S_c}\beta\ \Omega_N F_2(\tilde{\omega})\,dS$$

$$B_2 = \frac{\tilde{\omega} M - [\tilde{\omega}^2 M + (M^2-1)(\lambda^2_{\hat{\ell}\hat{m}} - \tilde{\omega}^2)]^{\frac{1}{2}}}{(1 - M^2)}$$

$$C = - \frac{e^{iLB_1}}{e^{iLB_2}} \frac{[B_1 + \beta_N(\gamma\tilde{\omega} + \gamma M B_1)]}{[B_2 + \beta_N(\gamma\tilde{\omega} + \gamma M B_2)]}$$

and ψ is determined by the condition

$$\iiint\limits_V \tilde{\phi}\ \Omega_{\hat{\ell}\hat{m}\hat{n}}\ dv = 1.$$

At the injector $\tilde{\phi}$ must satisfy the contition $\vec{\nabla}\tilde{\phi} \cdot \vec{n} = \beta_i p'$. For a given combustion response this equation serves as an equation for the determination of $\tilde{\omega}$. This equation may be solved for $\tilde{\omega}$ or, alternatively, Equations (1) and (2) may be solved for $\tilde{\omega}$ after β_c has been set equal to zero and $\tilde{\phi}$ has been substituted for ϕ. The latter method was used in this work.

Substitution of $\tilde{\phi}$ and $\tilde{\omega}$ into Equations (1) and (2) as the initial step in the iteration procedure described earlier eventually leads to the following two recussion formulas for the calculation of the j^{th} approximation for ϕ and ω

$$\phi^{(j)} = \tilde{\phi} + (\tilde{\omega}^2 - \omega^{2(j-1)}) \Big[\beta \int\limits_{S-S_c} H_N(\omega^{(j-1)}, \tilde{\omega}) F_2(\tilde{\phi}, \omega^{(j-1)})\ dS \Big]$$

$$+ M \int\limits_V H_N(\omega^{(j-1)}, \tilde{\omega}) F_1(\tilde{\phi}, \omega^{(j-1)})\ dV$$

$$+ i(\omega^{(j-1)} - \tilde{\omega}) \Big[\gamma \int\limits_{S-S_c} G_N(\tilde{\omega})\ \tilde{\phi} dS + 2M \underset{V}{\;} G_N(\tilde{\omega})\ \frac{\partial\tilde{\phi}}{\partial z}\ dV \Big]$$

After some quite extensive manipulation and integration it is possible to rewrite the expression for $\phi^{(j)}$ in the following form

$$\phi^{(j)} = \tilde{\phi} + \cos \hat{m}\theta \sum_{\ell} \sum_{n} \mu_{\ell n}^{(j)} J_{\hat{m}}(\lambda_{\ell n}\gamma) \cos \frac{n\pi\tau}{L}$$

$\mu_{\ell n}^{(j)}$ is a matrix of arbitrary dimensions.

For the calculations in this report ℓ went from 1 to 3 and n from 1 to 30. Larger matrices produced only slight increases in accuracy at great expense in computer time. Using the 3 x 30 matrix for $\mu_{\ell n}^{(j)}$ resulted in computation times of between 1 and 2 seconds per iteration on a CDC 6400 computer.

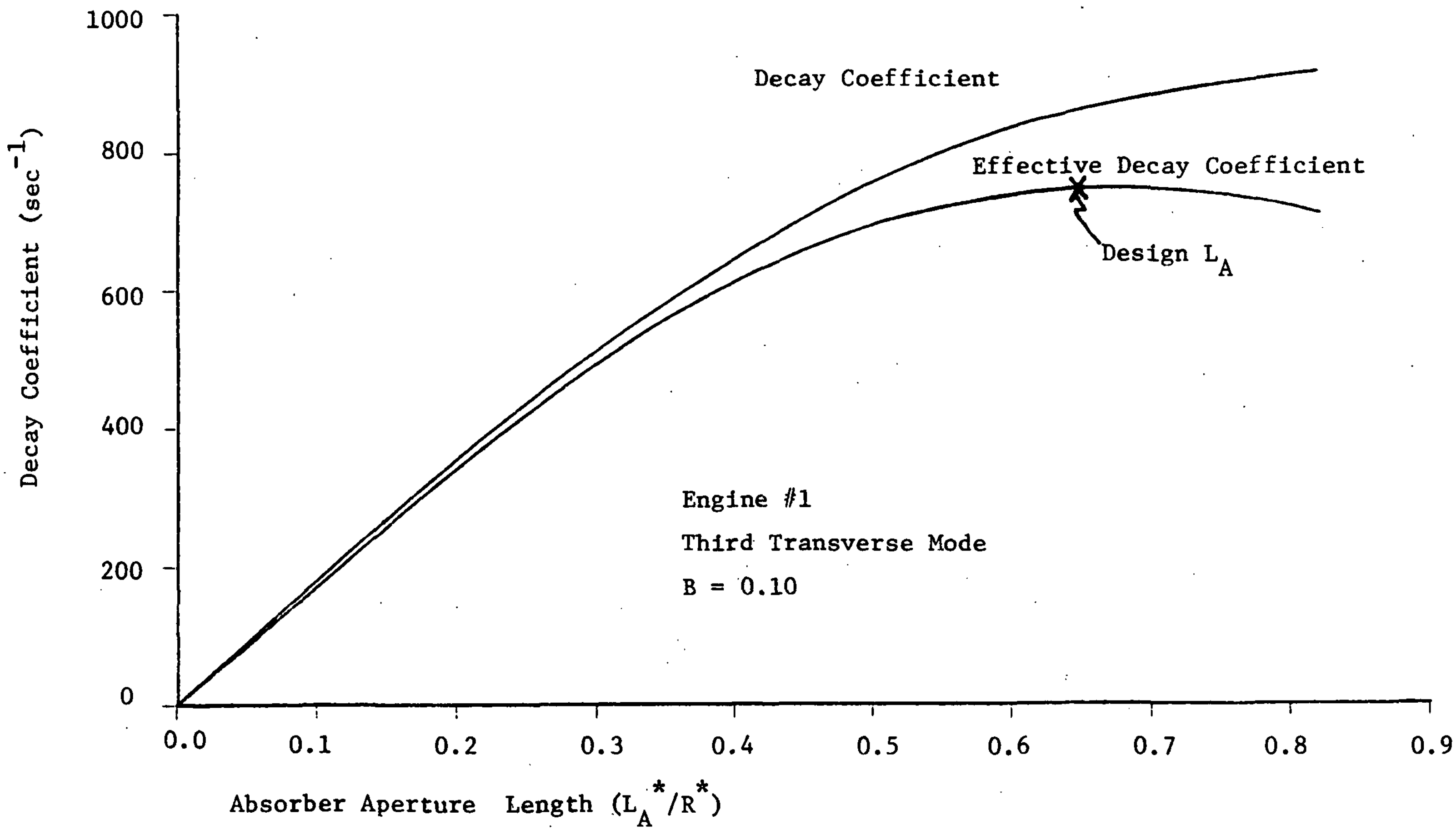


Figure A-1 Dependence of Decay Coefficient and Effective Decay Coefficient on Aperture Length.

APPENDIX B

Absorber Design

The following equation was used to calculate the impedance of the absorbers

$$K = \frac{0.1}{\gamma|K|} + i\left(\mathrm{Re}(\omega)\ \ell_{eff} - \frac{W_a}{2\mathrm{Re}(\omega)B^2}\right) \ldots . (B\text{-}1)$$

where $\ell_{eff} = L_a + 0.85\ W_a \left[1 - 0.7\sqrt{\frac{W_a}{2B}}\right]$, and $\mathrm{Re}(\omega)$ is the real part of the chamber frequency. Simple Helmholtz resonator theory was used to obtain this equation. (See for example Reference (5)). For a tuned liner the imaginary part of K is zero and the real part is equal to $\left(\frac{0.1}{\gamma}\right)^{\frac{1}{2}}$. Clearly, for a fixed backing distance, B, and aperture length L_a, the absorber can be tuned for a range of chamber frequencies by picking W_a properly (so that Im(K) = 0). Since the frequency and decay rate for the chamber are dependent not only on the absorber impedance but also on the aperture width, varying L_a for a given B will vary the chamber frequency and decay rate even though K remains fixed at its tuned value. For a given backing distance, then, frequencies and decay rates (and absorber aperture width) for tuned absorbers can be calculated as a function of L_a. Note that in these calculations $\mathrm{Re}(\omega)$ appears in the imaginary part of K. This dependency must, of course, be included in the iterative solution method for Equations (1) and (2) mentioned earlier.

For a given backing distance B there is some optimum aperture length L_a which will produce the maximum decay rate for the chamber. Unfortunately, for the range of backing distances and chamber configurations examined in this report the optimum L_a usually had a value approximately an order of

magnitude greater than the backing distance. Thus, for a backing distance of the order of one tenth the chamber radius optimum aperture lengths were of the order of the chamber radius. These aperture lengths were considered unrealistic.

In order to develop a better design an "effective decay coefficient", D_{eff}, was introduced defined by the following equation

$D_{eff} = 3D\left[1 + \left|\frac{K_2}{K_1}\right| + \left|\frac{K_3}{K_1}\right|\right]^{-1}$ where D is the decay coefficient for the chamber, K_1 is the impedance of the tuned absorber, K_2 is the impedance of the absorber at a frequency 1.5 times as large as the tuned frequency, and K_3 is the impedance at a frequency 2/3 as large as the tuned frequency. This effective decay coefficient serves as an approximate means of measuring the broad frequency band effectiveness of a given absorber design. For any absorber design the values of D_{eff} were always somewhat smaller than D. Moreover, optimum values of L_a in the sense of maximizing D_{eff} occurred at substantially lower values of L_a than did the optima in terms of maximum D. Still, the optimum L_a for maximum D_{eff} was usually too large for practical purposes and the arbitrary design condition $L_a = B$ was adopted in these cases. For third transverse and higher modes of oscillation the optimum L_a often occurred at a value less than B. When this happened, this smaller value of L_a was used in the absorber design. This latter type of behavior is shown in Figure (A-1), for the third transverse mode of oscillation in Engine 1 with a backing distance $B = 0.10$.

APPENDIX C
COMPUTER PROGRAMS

This section describes how the computer programs are used to determine the decay coefficient for a given set of input parameters which characterize the combustion chamber. The first program, COMBRES, calculates the neutral stability curves for the combustor. The second program, DECOEFF, uses the minimum interaction index which was determined in COMBRES to calculate the decay coefficient and absorber dimensions for a given backing distance.

These programs were written in FORTRAN IV to run on a CDC 6400. Many portions of the programs and the nomenclature in the programs are the same. When the programs were written very little effort was made to optimize the programs with respect to running time.

COMBRES

This program calculates three neutral stability curves for a given combustor for the combined first radial, first, second, and third transverse acoustic modes, respectively, as a function of frequency. A combustion response factor is calculated for each frequency and is then converted into the corresponding interaction index and sensitive time lag values. The frequency is incremented until a minimum interaction index is determined.

The input information required is:

1. LENGNO - the problem identifier
2. ALENGTH - the ratio of the length of the combustor to the radius of the combustor
3. AMACH - mean flow Mach Number

4. GAMMA - ratio of the specific heats

The output from this program includes the interaction index from which the minimum value can be determined. The combustion response factor corresponding to the minimum interaction index is then used as an input variable in DECOEFF.

DECOEFF

This program calculates the decay coefficient and absorber dimensions for a given backing distance of the absorber. This program uses the method of successive approximation to solve for the decay coefficient. A first guess at the frequency is calculated for the given combustion response factor. This frequency is used to calculate a new velocity potential and then a new frequency is calculated. This technique proceeds until the error between successive iterations is within the specified limits. The accuracy of the decay coefficient is limited only by the size of the coefficient matrix used in the calculation of the velocity potential.

The input parameters are:

1. LENGNO - the problem identifier
2. ALENGTH - the length to radius ratio for the combustor
3. AMACH - mean flow Mach Number
4. RADIUS - radius of the combustor in inches
5. SOS - speed of sound in the combustor
6. GAMMA - ratio of specific heats
7. FREQ - the real part of the frequency ratio for the minimum interaction index.

8. BETAI - combustion admittance corresponding to the minimum interaction index
9. MHAT - transverse acoustic mode number
10. NHAT - longitudinal acoustic mode number
11. MATRIX - identifier used to determine whether or not the coefficient matrix is printed out
12. B - ratio of the backing distance of the absorber to the radius of the combustor
13. ERROR1 - acceptable error in the real part of the frequency
14. ERROR2 - acceptable error in the imaginary part of the frequency
15. LDEX - specifies the number of terms in the radial direction to be calculated
16. NDEX - specifies the number of terms in the longitudinal direction to be calculated

The output from this program lists the input variables, the decay coefficient, and the absorber dimensions.

Sample Calculations

This section is an explanation of how the two programs are used to design an acoustic absorber and calculate the resulting decay coefficient, subject to the condition that the aperature length equals the backing distance. The data cards for each sample calculation are shown at the end of the appropriate program listing. On the page immediately following each program is the corresponding listing of the output of the sample calculation. For the sake of brevity only a partial listing of the output for COMBRES is given.

The first step in designing an acoustic absorber and calculating a decay coefficient for the absorber designed is to calculate the combustion admittance as a function of frequency. This determines the neutral stability map for the combustor when no absorber is present and determines the interaction index and sensitive time lag at the minimum of the stability curve. These neutral stability maps are calculated by COMBRES for the first, second, and third transverse acoustic modes. The form of the input for COMBRES is as follows:

Columns	Variable	Type
1-10	LENGNO	Alphanumeric word
11-20	ALENGTH	Real number
21-30	AMACH	Real number
31-40	GAMMA	Real number

On the page following the one displaying the above input data is a partial listing of the output from this program. From the lower half of the page the minimum for the interaction index, N, is determined by

inspection and the corresponding frequency ratio is noted, W/WO = 0.98. The frequency ratio is then used to locate the block of output containing the combustion admittance, upper half of page. In the output shown the frequency ratio is incremented by one-hundredth. This increment size may be changed by substituting the appropriate increment size on eleventh card from the bottom on page [F = F + CMPLX(increment, 0.0)]. It should be noted that there is an output block similar to that at the top of the page for each frequency ratio and that the output displayed is only for the first transverse acoustic mode. The output for the other transverse modes is similar to the above.

Once the minimum interaction index is determined to sufficient accuracy, the second step is to run DECOEFF to obtain the decay coefficient with respect to the combustion admittance previously calculated. In the sample run of DECOEFF given here the minimum n used was for W/WO = .982 with a corresponding value of β_i of 0.0761 + 0.06375i. These values are obtained when COMBRES is used with the increments for W/WO being in thousandths rather than in hundredths as in the sample above. The aperature width of the acoustic absorber is calculated according to the assumption that the aperature length equals the backing distance. The form of the input for DECOEFF is shown immediately following the list of the program. The input for the program in on two data cards as follows:

Card 1

Columns	Variable	Type
1-10	LENGNO	Alphanumeric word
11-20	ALENGTH	Real number
21-30	AMACH	Real number

31-40	RADIUS	Real number
41-50	SOS	Real number
51-60	GAMMA	Real number
61-70	W/WO	Real number

Card 2

Columns	Variable	Type
1-21	real part of BETAWI	Real number with exponent
22-42	imaginary part of BETAWI	Real number with exponent
43-44	MHAT	Integer number
45-46	NHAT	Integer number
47-50	MATRIX*	Alphanumeric number
51-55	B	Real number
56-60	ERROR1	Real number
61-65	ERROR2	Real number
66-70	LDEX	Integer number
71-75	NDEX	Integer number

The last page of the computer listing shows the form of the output of DECOEFF. The output includes most of the input variables, the decay coefficient, the effective decay coefficient, the absorber dimensions, and the location of the aperature opening with respect to the injector.

---

*If the coefficient matrix is to be printed out, then MATRIX is inputed as YESB, where B is a blank.

NOMENCLATURE

	DECOEFF	COMBRES
ω	OMEGA	OMEGA
ω	W	Not applicable
M	AMACH	AMACH
λ	GAMMA	GAMMA
η	Function ETA	Function ETA
$J'_m(\lambda_{\ell m})$	Function BESPRIM	Function BESPRIM
$J_m(\lambda_{\ell m})$	Function BESSEL	Function BESSEL
$\Lambda^{\frac{1}{2}}_{\ell mn}$	Function FNORM	Function FNORM
ψ	Function PSI	Function PSI
B_1	BETA1	BETA1
B_2	BETA2	BETA2
C	C	C
β_i	BETAI	BETAI
β_c	BETAC	BETAC
β_N	BETAN	BETAN
$\mu_{\ell n}$	MU	MU
K	AK	AK
L_{eff}	LEFF	Not applicable
L_A	AL or LA	Not applicable
W_A	WA	Not applicable
B	B	Not applicable

```
      PROGRAM COMBRES( OUTPUT=101, TAPE6=OUTPUT,INPUT=101,TAPE5=INPUT)
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2, N(150) , WWO(150)
      COMPLEX G, AN, AK, A, A4, A5, BETA, DUM1, DUM2, PSI, DUMMY, G1,
     1 G2,F
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, X, LHAT, MHAT, NHAT,
     1 PSI2
      K = 1
      READ(5,200)LENGNO,ALENGTH,AMACH,GAMMA
   25 GO TO (20,21,22,23),K
   20 MHAT = 1 : F = CMPLX(0.90, 0.0 ) : GO TO 24
   21 MHAT = 2 : F = CMPLX(1.50, 0.0 ) : GO TO 24
   22 MHAT = 3 : F = CMPLX(2.00, 0.0 ) : GO TO 24
   23 CALL EXIT
   24 K = K + 1
      I = 1
      X = 0.0
      BETAC = CMPLX(0.0,0.0)
    7 CONTINUE
      WWO(I) = F
      A = CMPLX ( 1.0 , 0.0 )
      G = CMPLX( 0.9166, 0.0 )
      BETAN = AMACH*(G-1./GAMMA)
      LHAT = 1
      NHAT=0
      RLMDA = BESPRIM ( MHAT + 1 , LHAT )
      NHAI = NHAT + 1
      OMEGA=F*1.84118378
      BETA=CSQRT(OMEGA*AMACH*OMEGA*AMACH+(AMACH*AMACH-1.)*(RLMDA**2 -
     1OMEGA*OMEGA))/(1.-AMACH*AMACH)
      BETA1=(OMEGA*AMACH)/(1.-AMACH*AMACH)+BETA
      BETA2=(BETA1)-2.*BETA
      A4 = CMPLX( -AIMAG(BETA1), REAL(BETA1))
      DUM1 = CMPLX( -AIMAG(BETA2), REAL(BETA2))
      C = - CEXP(ALENGTH*A4) / CEXP(ALENGTH*DUM1)
      C = C * (BETA1 + BETAN * GAMMA * (OMEGA + AMACH * BETA1 )) / (
     1 BETA2 + BETAN * GAMMA * (OMEGA + AMACH * BETA2 ))
      PSI2 = PSI( DUMMY )**2
      AK1=(OMEGA+      AMACH *BETA1+C*(OMEGA+      AMACH*BETA2))*GAMMA
      AK1= CMPLX(-AIMAG(AK1),REAL(AK1))
      DUM1 = CMPLX(-AIMAG(BETA1),REAL(BETA1))
      DUM1 = DUM1*ALENGTH
```

```
      DUM2 = CMPLX(-AIMAG(BETA2),REAL(BETA2))
      DUM2 = DUM2*ALENGTH
      AK2 = ((OMEGA+AMACH*BETA1)*CEXP(DUM1)+(OMEGA+AMACH*BETA2)*C*CEXP(
     1DUM2))*GAMMA
      AK2 = CMPLX(-AIMAG(AK2),REAL(AK2))
      CALL CALCA4( A4, A5 )
      BETAWI = (OMEGA**2 - ETA(LHAT,MHAT,NHAT,ALENGTH,RLMDA)-(AMACH*G2(
     1NHAT)/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)+BETAN*AK2/EPSIZN(LHAT,
     1MHAT,NHAT,ALENGTH,RLMDA)*(-1.)**(NHAT+2))/PSI(DUMMY))/A4
      AN = 1. / GAMMA - BETAWI / AMACH
      IF ( REAL ( AN ) .LT. 0.00 ) GO TO 1
      N(I) = AN
      WRITE(6,100)LENGNO
      WRITE(6,116) AMACH
      WRITE(6,117) GAMMA
      WRITE(6,118) OMEGA
      WRITE(6,119) ALENGTH
      WRITE(6,101) LHAT, MHAT, NHAT
      WRITE(6,115) X
      WRITE(6,113) F
      WRITE(6,114) A
      WRITE(6,102) BETAN
      WRITE(6,120) G
      WRITE(6,121) BETAC
      WRITE(6,108)
      WRITE(6,105) BETAWI
      WRITE(6,103) AN
      BETAWI = ( BETA1 + C * BETA2 ) / ( GAMMA * (OMEGA * ( 1. + C ) +
     1 AMACH * ( BETA1 + C * BETA2 ) ) )
      WRITE(6,112) BETAWI
      AN = 1. / GAMMA - BETAWI / AMACH
      WRITE(6,103) AN
      I = I + 1 : F = F + CMPLX ( 0.01 , 0.0 )
      IF(REAL(AN) .LT.1.8 .AND. AIMAG(AN) .LT. 1.0 ) GO TO 7
  500 CALL NTAU(AMACH,REAL(AK),N,WW0,I-1)
      CALL EXIT
      GO TO 25
    1 F = F + CMPLX(0.01, 0.0 )
      GO TO 7
  100 FORMAT('1 THIS IS ENGINE NUMBER 'A10)
  101 FORMAT('0 THE PRIMARY MODE ASSUMED IS LHAT = ',I2,' MHAT = ',I2,'
     1NHAT = ',I2)
  102 FORMAT('0 BETAN = ',2G21.14)
```

```
  103 FORMAT('+',70X,' N = ',2G21.14,'I')
  104 FORMAT(' ',I3,'  ',10G13.6)
  105 FORMAT('0 BETAWI = ',2G21.14)
  106 FORMAT('0 N             J EQUAL ',I2,'      L  EQUAL ',I1,' TO ',I2)
  108 FORMAT('+',70X,' K = INFINITY')
  109 FORMAT('0 BETAI(',I2,') = ',2G21.14)
  112 FORMAT('0 BETAWID = ',2G21.14)
  113 FORMAT('0 W/W0 = ',2G21.14)
  114 FORMAT('+',70X,' A = ',2G21.14)
  115 FORMAT('+',70X,' THE LENGTH OF THE LINER IS ',G21.14)
  116 FORMAT('0 THE MACH NUMBER IS ',G21.14)
  117 FORMAT('+',70X,' THE RATIO OF SPECIFIC HEATS IS ',G21.14)
  118 FORMAT('0 THE FREQUENCY IS ', 2G21.14)
  119 FORMAT('+',70X,' THE LENGTH OF THE COMBUSTOR IS ',G21.14)
  120 FORMAT('+',70X,' G = ', 2G21.14)
  121 FORMAT('0 BETAC = ', 2G21.14)
  200 FORMAT(A10,3F10.0)
      END
      FUNCTION BESPRIM (M, L)
      DIMENSION A(10,5)
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,1) = 0.00000000
      A(2 ,1) = 3.83170597
      A(3 ,1) = 7.01558667
      A(4 ,1) = 10.17346814
      A(5 ,1) = 13.32369194
      A(6 ,1) = 16.47063005
      A(7 ,1) = 19.61585851
      A(8 ,1) = 22.76008438
      A(9 ,1) = 25.90367209
      A(10,1) = 29.04682853
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,2) = 1.84118378
      A(2 ,2) = 5.33144277
      A(3 ,2) = 8.53631637
      A(4 ,2) = 11.70600490
      A(5 ,2) = 14.86358863
      A(6 ,2) = 18.01552786
      A(7 ,2) = 21.16436986
      A(8 ,2) = 24.31132686
      A(9 ,2) = 27.45705057
```

```
      A(10,2) = 30.60192297
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,3) = 3.05423693
      A(2 ,3) = 6.70613319
      A(3 ,3) = 9.96946782
      A(4 ,3) = 13.17037086
      A(5 ,3) = 16.34752232
      A(6 ,3) = 19.51291278
      A(7 ,3) = 22.67158177
      A(8 ,3) = 25.82603714
      A(9 ,3) = 28.97767277
      A(10,3) = 32.12732702
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,4) = 4.20118894
      A(2 ,4) = 8.01523660
      A(3 ,4) = 11.34592431
      A(4 ,4) = 14.58584829
      A(5 ,4) = 17.78874787
      A(6 ,4) = 20.97247694
      A(7 ,4) = 24.14489743
      A(8 ,4) = 27.31005793
      A(9 ,4) = 30.47026881
      A(10,4) = 33.62694918
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,5) = 5.31755313
      A(2 ,5) = 9.28239629
      A(3 ,5) = 12.68190844
      A(4 ,5) = 15.96410704
      A(5 ,5) = 19.19602880
      A(6 ,5) = 22.40103227
      A(7 ,5) = 25.58975968
      A(8 ,5) = 28.76783622
      A(9 ,5) = 31.93853934
      A(10,5) = 35.10391668
    1 BESPRIM = A(L, M)
      RETURN
      ENTRY BESSEL
C**** THESE ARE THE BESSEL NUMBERS OF ORDER ZERO  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,1) = 1.00000000
```

```
      A(2 ,1) = -0.4027588095
      A(3 ,1) = 0.301128303
      A(4 ,1) = -0.249704877
      A(5 ,1) = 0.218359407
      A(6 ,1) = -0.19645371
      A(7 ,1) = 0.180063375
      A(8 ,1) = -0.167184600
      A(9 ,1) = 0.156724985
      A(10,1) = -0.148011108
C**** THESE ARE THE BESSEL NUMBERS OF ORDER  ONE  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,2) = 0.5818649368
      A(2 ,2) = -0.3461258542
      A(3 ,2) = 0.2732981131
      A(4 ,2) = -0.233304416
      A(5 ,2) = 0.207012651
      A(6 ,2) = -0.188017488
      A(7 ,2) = 0.173459050
      A(8 ,2) = -0.161838211
      A(9 ,2) = 0.152282069
      A(10,2) = -0.144242905
C**** THESE ARE THE BESSEL NUMBERS OF ORDER  TWO  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,3) = 0.4864961885
      A(2 ,3) = -0.3135283099
      A(3 ,3) = 0.2547441235
      A(4 ,3) = -0.220881581
      A(5 ,3) = 0.197937434
      A(6 ,3) = -0.181010000
      A(7 ,3) = 0.167835534
      A(8 ,3) = -0.157195167
      A(9 ,3) = 0.148363778
      A(10,3) = -0.140878333
C**** THESE ARE THE BESSEL NUMBERS OF ORDER THREE FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,4) = 0.4343942763
      A(2 ,4) = -0.2911584413
      A(3 ,4) = 0.240738175
      A(4 ,4) = -0.210965204
      A(5 ,4) = 0.190419022
      A(6 ,4) = -0.175048405
      A(7 ,4) = 0.162954965
      A(8 ,4) = -0.153102409
```

```
      A(9 ,4) = 0.144866574
      A(10,4) = -0.137844513
C**** THESE ARE THE BESSEL NUMBERS OF ORDER FOUR  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,5) = 0.3996514545
      A(2 ,5) = -0.2743809949
      A(3 ,5) = 0.229590468
      A(4 ,5) = -0.202763849
      A(5 ,5) = 0.184029896
      A(6 ,5) = -0.169878516
      A(7 ,5) = 0.158655372
      A(8 ,5) = -0.149451156
      A(9 ,5) = 0.141714307
      A(10,5) = -0.135086328
      GO TO 1
      END
      SUBROUTINE CALCA4( A4, A5 )
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2
      COMPLEX A4, A5, PSI, DUMMY, G1, G2
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, X, LHAT, MHAT, NHAT,
     1 PSI2
      A4 = AK1/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)/PSI(DUMMY)
      A5=BETAC*G1(NHAT)/EPSIKAT(LHAT,MHAT,NHAT,ALENGTH,RLMDA)/PSI(DUMMY)
      RETURN
      END
      FUNCTION EPSIKAT(L,M,N,ALENGTH,RLMDA)
      DUM1 = ALENGTH
      IF( N    .NE. 0) DUM1 = DUM1/2.
      IF( M    .EQ. 0) GOTO 1
      EPSIKAT = (0.5-M   *M   /(2.*RLMDA*RLMDA))*DUM1
      RETURN
    1 EPSIKAT = DUM1/2.
      RETURN
      ENTRY EPSIZN
      IF( N .EQ. 0 ) GO TO 2
      EPSIKAT= ALENGTH / 2.
      RETURN
    2 EPSIKAT= ALENGTH
      RETURN
      ENTRY ETA
      EPSIKAT = (N-1)*(N-1) * 3.1415926 *3.1415926 / (ALENGTH * ALENGTH)
```

```
1+RLMDA * RLMDA
 RETURN
 END
 COMPLEX FUNCTION G1 ( N )
 COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
1 AK2, PSI2
 COMPLEX DUM1, GDUM, BETA, GBUM, DUM
 COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, X, LHAT, MHAT, NHAT,
1 PSI2
 GDUM(A,DUM1,BETA,OMEGA,AMACH,X) = (OMEGA+AMACH*BETA)/(A**2-BETA**2
1 )*(CEXP(DUM1*X)*(DUM1*COS(A*X)+A*SIN(A*X))-DUM1)
 GBUM(BETA,DUM1,N,AMACH,OMEGA,ALENGTH) = -(AMACH*BETA*BETA+2.*OMEGA
1 *BETA)/(( N * 3.1415926 / ALENGTH )**2 - BETA**2 ) *(CEXP(DUM1*
1 ALENGTH) * DUM1 * (-1.)**(N+2) - DUM1 )
 DUM(DUM1,ALENGTH,NHAT) = CEXP(DUM1*ALENGTH)*(-1.)**(NHAT+2)-1.
 IF( X .EQ. 0.0 ) GO TO 1
 A = N*3.1415926/ALENGTH
 BETA  = BETA1
 DUM1 = CMPLX(-AIMAG(BETA1),REAL(BETA1))
 G1 = GDUM(A,DUM1,BETA,OMEGA,AMACH,X)
 IF( N .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) GO TO 2
 BETA = BETA2
 DUM1 = CMPLX(-AIMAG(BETA2),REAL(BETA2))
 G1 = GAMMA * (G1 + C * GDUM(A,DUM1,BETA,OMEGA,AMACH,X))
 G1 = CMPLX( -AIMAG(G1),REAL(G1))
 RETURN
1 G1 = CMPLX(0.0,0.0)
 RETURN
2 BETA = C * OMEGA * X
 G1 = GAMMA * ( G1 + BETA )
 G1 = CMPLX( -AIMAG(G1), REAL(G1))
 RETURN
 ENTRY G2
 BETA = BETA1
 DUM1 = CMPLX(-AIMAG(BETA1),REAL(BETA1))
 G1 = GBUM(BETA,DUM1,N,AMACH,OMEGA,ALENGTH)
 IF( N .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) RETURN
 BETA = BETA2
 DUM1 = CMPLX(-AIMAG(BETA2),REAL(BETA2))
 G1 = G1 + C * GBUM(BETA,DUM1,N,AMACH,OMEGA,ALENGTH)
 RETURN
 ENTRY PSI
```

```
      DUM1 = CMPLX( -AIMAG(BETA1), REAL(BETA1))
      G1  = DUM1*DUM(DUM1,ALENGTH,NHAT)/(NHAT**2*3.1415926**2/ALENGTH**2
     1 -BETA1**2)
      IF( NHAT .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) GO TO 3
      DUM1 = CMPLX( -AIMAG(BETA2), REAL(BETA2))
      G1  = ( G1 +C*DUM1*DUM(DUM1,ALENGTH,NHAT)/(NHAT**2*3.1415926**2/
     1 ALENGTH**2-BETA2**2))/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)
      RETURN
    3 G1  = ( G1  + C * ALENGTH ) / EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)
      RETURN
      END
      SUBROUTINE NTAU(MACH,K,AN,WWO,I)
      COMPLEX AN(1) , WWO(1)
      REAL N, MACH, K, LINER
      WRITE(6,102)
      PIE2 = 2. * 3.14159265358979323846
      DO 7 J = 1,I
      ANR = REAL(AN(J))
      ANI = AIMAG(AN(J))
      O = REAL(WWO(J))
      OMEGA = O * 1.84118378
      N = ( ANR**2 + ANI**2 ) / 2. / ANR
      IF ( ANI .LT. 0.0 ) 5, 6
    5 TAU = (PIE2-ACOS(1.-ANR/N))/OMEGA
      GO TO 7
    6 TAU = ACOS( 1.-ANR / N ) / OMEGA
    7 WRITE(6,101) MACH, O, OMEGA, N, TAU
  101 FORMAT('0',5X,F5.3,5X,F5.2      ,5X,G18.11,2(5X,G21.14))
  102 FORMAT('1',6X,'MACH',6X,'W/W0',9X,'OMEGA',20X,'N',24X,'TAU')
      RETURN
      END
```

THE FOLLOWING IS AN EXAMPLE OF THE PUNCHED CARD INPUT.

```
ONE       1.984159  0.265865  1.146
```

```
THIS IS ENGINE NUMBER ONE
THE MACH NUMBER IS    .26586500000000
THE RATIO OF SPECIFIC HEATS IS    1.1460000000000
THE FREQUENCY IS    1.8043601044000      0.
THE LENGTH OF THE COMBUSTOR IS    1.9841590000000
THE PRIMARY MODE ASSUMED IS LHAT =  1 MHAT =  1 NHAT =  0
THE LENGTH OF THE LINER IS  0.
W/W0 =    .97999999999998      0.
A =    1.0000000000000      0.
BETAN =  1.16979672024435E-02 0.
G =    .91660000000000      0.
BETAC =  0.                     0.
K = INFINITY
BETAWI =  8.25323501414754E-02 7.00912836202505E-02
N =    .56217080719944      -.26363486589153    1
BETAWID =  8.25323501414719E-02 7.00912836202479E-02
N =    .56217080719946      -.26363486589152    1
```

MACH	W/W0	OMEGA	N	TAU
.266	.90	1.66	20.526968	3.7584663
.266	.91	1.68	7.4276621	3.6665508
.266	.92	1.69	3.7860679	3.5616498
.266	.93	1.71	2.1566187	3.4374212
.266	.94	1.73	1.2912086	3.2845350
.266	.95	1.75	.80152975	3.0904435
.266	.96	1.77	.52708698	2.8429163
.266	.97	1.79	.38891625	2.5435533
.266	.98	1.80	.34290233	2.2271651
.266	.99	1.82	.36160715	1.9513585
.266	1.00	1.84	.42616618	1.7525075
.266	1.01	1.86	.52232427	1.6324354
.266	1.02	1.88	.63840129	1.5780414
.266	1.03	1.90	.76426740	1.5756145
.266	1.04	1.91	.89090300	1.6137839
.266	1.05	1.93	1.0103177	1.6820456

```
      PROGRAM DECOEFF(INPUT=101,OUTPUT=101,TAPE5=INPUT,TAPE6=OUTPUT)
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2, MU, W, F, I,SOME,SOME1,SG1,SSG1,Y,Z
      COMPLEX G, AN, AK, A, A4, A5, BETA, DUM1, DUM2, PSI, DUMMY, G1,
     1 G2, WI ,UHAT
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, I, LHAT, MHAT, NHAT,
     1 PSI2 ,PI,X2,X1,SOME,SOME1
      COMMON / BLKB / MU( 3,30,29) , W(30)
      COMMON / BLK1 / F,RHOCHAM,RHOVOL,PRESURE,RADIUS,SOS
      COMMON C11(30,30) , C10(30,30) , ERROR1 , ERROR2 , LDEX , NDEX
    8 CONTINUE
      I = CMPLX(0.0 ,1.0 )
      PI = 3.14159265358974
      A = CMPLX ( 1.0 , 0.0 )
      G = CMPLX( 0.9166, 0.0 )
      READ(5,200)LENGNO,ALENGTH,AMACH,RADIUS,SOS,GAMMA,FREQ,BETAI,MHAT,
     1NHAT,MATRIX,B,ERROR1,ERROR2,LDEX,NDEX
      IF ( EOF,5 ) 9 , 10
   10 RAD = RADIUS
      AN = 1. / GAMMA - BETAI / AMACH
      RADIUS = RADIUS/12.0
      F = CMPLX ( FREQ , 0.0 )
    4 W = REAL(F)*1.84118378
      AP = 0.595*0.595/2./B
      BP = 0.85-0.5/(W*B)**2
      CALL CUBIC(-BP*BP/AP,-2.*B*BP/AP,-B*B/AP,B,WA)
      CALL ABSORB(AN,B,WA,AL,AK,UHAT,GAMMA)
      FREQ1 = REAL(F)
      X1 = ( B-WA/2.)
      X2 = X1 + WA
      BETAC = 1./AK       /GAMMA
      BETAN = AMACH * ( G - 1. / GAMMA )
      LHAT = 1
      RLMDA = BESPRIM ( MHAT + 1 , LHAT )
    7 CONTINUE
      NHAI = NHAT + 1
      F= CMPLX ( FREQ , 0.0 )
      OMEGA = CMPLX ( 1.84118378 , 0.0 )        * F
      DO 2 IK= 1, 500
      BETA=CSQRT(OMEGA*AMACH*OMEGA*AMACH+(AMACH*AMACH-1.)*(RLMDA**2 -
     1OMEGA*OMEGA))/(1.-AMACH*AMACH)
      BETA1=(OMEGA*AMACH)/(1.-AMACH*AMACH)+BETA
```

```
      BETA2=(BETA1)-2.*BETA
      C = - CEXP(ALENGTH*I*(BETA1- BETA2))
      C = C * (BETA1 + BETAN * GAMMA * (OMEGA + AMACH * BETA1 )) / (
     1 BETA2 + BETAN * GAMMA * (OMEGA + AMACH * BETA2 ))
      PSI2 = PSI( DUMMY )**2
      AK1=(OMEGA+      AMACH *BETA1+C*(OMEGA+      AMACH*BETA2))*GAMMA*I
      DUM1 = BETA1*I*ALENGTH
      DUM2 = BETA2*I*ALENGTH
      AK2 = ((OMEGA+AMACH*BETA1)*CEXP(DUM1)+(OMEGA+AMACH*BETA2)*C*CEXP(
     1DUM2))*GAMMA*I
      WI = ((BETAN*AK2*COS(NHAT*3.14159265358974)+BETAI*AK1+AMACH*G2(NHA
     1T))/PSI(NHAT)/EPSIZN(L,M,NHAT,ALENGTH,RLMDA)+ETA(L,M,NHAI,ALENGTH,
     1RLMDA))
      WI = CSQRT(WI)
      IF ( REAL ( WI ) .LT. 0.0 ) WI = - WI
      PSI2 = PSI ( DUMMY )
      IF ( CABS(WI-OMEGA).LT.1.E-07) 3 , 2
    2 OMEGA = WI
      WRITE(6,125)
      CALL EXIT
    3 OMEGA = WI
      F = WI / 1.84118378
      CALL CALCA4( A4, A5 )
      BETAWI = (OMEGA**2 - ETA(LHAT,MHAT,NHAI,ALENGTH,RLMDA)-(AMACH*G2(
     1NHAT)/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)+BETAN*AK2/EPSIZN(LHAT,
     1MHAT,NHAT,ALENGTH,RLMDA)*(-1.)**(NHAT+2))/PSI(DUMMY))/A4
      WRITE(6,100) LENGNO
      WRITE(6,116) AMACH
      WRITE(6,117) GAMMA
      WRITE(6,118) OMEGA
      WRITE(6,119) ALENGTH
      WRITE(6,101) LHAT, MHAT, NHAT
      WRITE(6,115) X1 , X2
      WRITE(6,113) F
      WRITE(6,114) A
      WRITE(6,102) BETAN
      WRITE(6,120) G
      WRITE(6,121) BETAC
      WRITE(6,108) AK
      WRITE(6,105) BETAWI
      AN = 1. / GAMMA - BETAWI / AMACH
      WRITE(6,103) AN
      BETAWI = ( BETA1 + C * BETA2 ) / ( GAMMA * (OMEGA * ( 1. + C ) +
```

```
     1 AMACH * ( BETA1 + C * BETA2 ) ) )
      WRITE(6,112) BETAWI
      AN = 1. / GAMMA - BETAWI / AMACH
      WRITE(6,103) AN
      AN = 1. / GAMMA - BETAI / AMACH
      WRITE(6,122) BETAI
      WRITE(6,103) AN
      WRITE(6,126) OMEGA , F
      WRITE(6,130) B , WA , UHAT , AL
      WRITE(6,131) FREQ1
      IF ( ABS(AIMAG(BETAC))  .GT. 1.E-05 ) GO TO 1
      WI=(BETAC*(G1(X2,NHAT)-G1(X1,NHAT))/PSI2  /EPSIKAT(LHAT,MHAT,NHAT,
     1ALENGTH,RLMDA))+OMEGA**2
      Z=WI
      NN=0
      WI = CSQRT(WI)
      IF(REAL(WI).LE.0.0)WI = -WI
      Y=WI
      SOME=SG1(X2,NHAT) - SG1(X1,NHAT)
      SOME1=SSG1(ALENGTH,NHAT)
      SOME=(((BETAI*(1. +C) + BETAN*(-1.)**NHAT*(CEXP(DUM1) + C*CEXP(
     1DUM2)))*I*GAMMA +2.*I*AMACH*SOME1)/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,
     2RLMDA) + I* GAMMA*BETAC*SOME/EPSIKAT(LHAT,MHAT,NHAT,ALENGTH,RLMDA)
     3)/PSI2
   14 CONTINUE
      WI=Z +(Y-OMEGA)*SOME
      WI=CSQRT(WI)
      IF(REAL(WI).LE.0.0) WI=-WI
      IF(CABS(WI - Y).LT.1.E-07.OR.NN.GT.10) 12,13
   13 Y=WI
      NN=NN +1
      GOTO 14
   12 W(1)=WI
      F = WI / 1.84118378
      WRITE(6,109) WI
      WRITE(6,129) F
      JX = 30
      JY = JX - 1
      CALL CALMU( JY )
      JX = JY + 1
      IF ( JY .EQ. 29 ) WRITE(6,125)
      CALL ABSORB(AN,B,WA,AL,AK,UHAT,GAMMA)
      WRITE(6,130) B , WA , UHAT , AL
```

```
      DAMP = AIMAG(W(JX))
      DSTAR = DAMP*SOS/RADIUS
      WRITE(6,107)DAMP , DSTAR
      CALL DEFFEC ( W(JX),B,WA,AL,GAMMA,AK,RADIUS,SOS,RLMDA)
      WRITE(6,123) RAD , RADIUS
      IF ( MATRIX .NE. 3HYES ) GO TO 1
      DO 6 J = 1, JY
      KS = 1
      KF=3
      WRITE(6,106) J, KS, KF
      DO6 L=1,30
    6 WRITE(6,104) L, (  MU(I,L,J), I = KS, KF )
    1 CONTINUE
      GO TO 8
    9 CALL EXIT
  100 FORMAT('1 THIS IS ENGINE NUMBER ',A10)
  101 FORMAT('0 THE PRIMARY MODE ASSUMED IS LHAT = ',I2,' MHAT = ',I2,'
     1NHAT = ',I2)
  102 FORMAT('0 BETAN = ',2G21.14)
  103 FORMAT('+',70X,' N = ',2G21.14,'I')
  104 FORMAT(' ',I3,'  ',10G13.6)
  105 FORMAT('0 BETAWI = ',2G21.14)
  106 FORMAT('0 N             J EQUAL ',I2,'       L  EQUAL ',I1,' TO ',I2)
  107 FORMAT('0 THE DAMPING RATE IS 'G21.14,28X' THE LOG DAMPING RATE IS
     1 'G21.14' INVERSE SECONDS')
  108 FORMAT('+',70X,' K = ',2G21.14)
  109 FORMAT('0 OMEGA( 1) = '2G21.14)
  112 FORMAT('0 BETAWID = ',2G21.14)
  113 FORMAT('0 W/W0 = '2G21.14)
  114 FORMAT('+',70X,' A = ',2G21.14)
  115 FORMAT('+',70X,' THE LINER STARTS AT 'F6.4' AND ENDS AT 'F6.4)
  116 FORMAT('0 THE MACH NUMBER IS ',G21.14)
  117 FORMAT('+',70X,' THE RATIO OF SPECIFIC HEATS IS ',G21.14)
  118 FORMAT('0 THE FREQUENCY IS ', 2G21.14)
  119 FORMAT('+',70X,' THE LENGTH OF THE COMBUSTOR IS ',G21.14)
  120 FORMAT('+',70X,' G = ', 2G21.14)
  121 FORMAT('0 BETAC = ', 2G21.14)
  122 FORMAT('0 IMPEDANCE OF THE INJECTOR IS '2F18.14)
  123 FORMAT('0 THE RADIUS IS 'F10.5' INCHES OR 'F10.5' FEET')
  125 FORMAT('1',135(1H:),//,'       THE PROBLEM DID NOT CONVERGE    ',//,1
     135(1H*))
  126 FORMAT('0 OMEGA WIGGLEY = '2G21.14,11X,' W/W0 WIGGLEY = '2G21.14)
  129 FORMAT('+'70X' W/W0 = '2G21.14)
```

```
130 FORMAT('0 THE BACKING DISTANCE IS ' G21.14,24X,' THE WIDTH OF THE
   1APERATURE IS 'G21.14//'  THE PEAK VELOCITY IS '2G21.14,6X,' THE LE
   1NGTH OF THE APERATURE IS ' G21.14)
131 FORMAT('0 THE DIMENSIONAL FREQUENCY IS 'G21.14)
200 FORMAT(A10,6F10.0/2G21.14,2I2,A4,3F5.0,2I5)
    END
    SUBROUTINE ABSORB(A,B,WA,LA,Z,UHAT,GAMMA)
    REAL LA , LEFF
    COMPLEX A, F , Z , UHAT
    COMMON / BLK1 / F,RHOCHAM,RHOVOL,PRESURE,RADIUS,SOS
    W = 1.84118378*REAL(F)
    LEFF = WA / 2. / W / W / B / B
    LA = LEFF - 0.85 * WA * ( 1.-0.7*SQRT(WA/2./B))
    IF ( LA .GE. 0.001 / RADIUS ) 2 , 1
  1 LA = 0.001 / RADIUS
    XA = W * LEFF
    XC = WA / 2. / W / B / B
    ZI = XA - XC
    ZR = SQRT(-ZI*ZI+SQRT(ZI*ZI*ZI*ZI+0.04/GAMMA/GAMMA))/SQRT(2.0)
    Z = CMPLX( ZR , ZI )
    GO TO 3
  2 Z = CMPLX ( SQRT(0.1/GAMMA) , 0.0 )
  3 UHAT = CMPLX ( 0.1 / CABS(Z) , 0.0 )
    F = CMPLX ( W , 0.0 )
    RETURN
    END
    SUBROUTINE DEFFEC ( W,B,WA,LA,GAMMA,Z1,RADIUS,SOS,RLMDA)
    REAL LA,LEFF,KR,KI
    COMPLEX W , Z1 , Z2, Z3
    KR ( ZI , GAMMA ) = SQRT(-ZI*ZI+SQRT(ZI*ZI*ZI*ZI+0.04/GAMMA/GAMMA)
   1)/SQRT(2.0)
    KI ( W1, LEFF , WA , B ) = W1*LEFF-WA/2./W1/B/B
    LEFF = LA + 0.85*WA*(1.0-0.7*SQRT(WA/B))
    WR = REAL(W)
    WI = AIMAG(W)
    ZI = KI(WR*1.5,LEFF,WA,B)
    Z2 = CMPLX ( KR(ZI,GAMMA),ZI)
    ZI = KI(WR/1.5,LEFF,WA,B)
    Z3 = CMPLX( KR(ZI,GAMMA), ZI)
    DAMP = WI*3.0/(1.0+(CABS(Z2)+CABS(Z3))/CABS(Z1))
    DSTAR = DAMP*SOS/RADIUS
    WRITE(6,100) DAMP , DSTAR
100 FORMAT('0 THE EFFECTIVE DAMPING RATE IS 'G21.14,18X' THE EFFECTIVE
```

```
     1 L. D. R. IS 'G21.14' INVERSE SECONDS')
      RETURN
      END
      SUBROUTINE CUBIC(P,Q,R,B,DELSTAR)
      AX=(3.*Q-P*P)/3.
      BX=(2.*P*P*P-9.*P*Q+27.*R)/27.
      AX1= AX * AX * AX / 27.
      BX1= BX * BX / 4.
      DIS = AX1+ BX1
      IF ( ABS ( DIS / AX1 ) .LE. 1.E-07 ) DIS = 0.0
      IF ( DIS .LT. 0.0 ) GO TO 1
      IF(DIS)1,2,2
    1 PHI=(ACOS(-BX/2./SQRT(-AX*AX*AX/27.)))/3.
      XCOEFF=2.*SQRT(-AX/3.)
      CON=0.017453292519943
      X1=XCOEFF*COS(PHI)-P/3.
      X2=XCOEFF*COS(PHI+120.*CON)-P/3.
      X3=XCOEFF*COS(PHI+240.*CON)-P/3.
      IF(X1.LT.0.0.OR.X1.GT. B )X1=-1000.
      IF(X2.LT.0.0.OR.X2.GT. B )X2=-1000.
      IF(X3.LT.0.0.OR.X3.GT. B )X3=-1000.
      DELSTAR=AMAX1(X1,X2,X3)
      GO TO 4
    2 EXP=1./3.
      BX2=-BX/2.
      C1=BX2+SQRT(DIS)
      C2=BX2-SQRT(DIS)
      ICODE=0
      JCODE=0
      IF(C1.LT.0.0)ICODE=1
      IF(C2.LT.0.0)JCODE=1
      AXE=(ABS(C1))**EXP
      BXE=(ABS(C2))**EXP
      IF(ICODE)7,8,7
    7 AXE=-AXE
    8 IF(JCODE)9,10,9
    9 BXE=-BXE
   10 IF(DIS)11,11,3
   11 X1=AXE+BXE-P/3.
      X2=-X1/2.-P/2.
      IF(X1.LT.0.0.OR.X1.GT. B )X1=-1000.
      IF(X2.LT.0.0.OR.X2.GT. B )X2=-1000.
      DELSTAR=AMAX1(X1,X2)
```

```
      GO TO 4
    3 DELSTAR=AXE+BXE-P/3.
    4 IF(DELSTAR.LE.0.0.OR.DELSTAR.GE. B )GO TO 5
      RETURN
    5 WRITE(6,6)
      CALL EXIT
    6 FORMAT(1H1,'                    THIS VALUE IS OUT OF RANGE.')
      END
      FUNCTION BESPRIM (M, L)
      DIMENSION A(10,5)
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,1) = 0.00000000
      A(2 ,1) = 3.83170597
      A(3 ,1) = 7.01558667
      A(4 ,1) = 10.17346814
      A(5 ,1) = 13.32369194
      A(6 ,1) = 16.47063005
      A(7 ,1) = 19.61585851
      A(8 ,1) = 22.76008438
      A(9 ,1) = 25.90367209
      A(10,1) = 29.04682853
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,2) = 1.84118378
      A(2 ,2) = 5.33144277
      A(3 ,2) = 8.53631637
      A(4 ,2) = 11.70600490
      A(5 ,2) = 14.86358863
      A(6 ,2) = 18.01552786
      A(7 ,2) = 21.16436986
      A(8 ,2) = 24.31132686
      A(9 ,2) = 27.45705057
      A(10,2) = 30.60192297
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,3) = 3.05423693
      A(2 ,3) = 6.70613319
      A(3 ,3) = 9.96946782
      A(4 ,3) = 13.17037086
      A(5 ,3) = 16.34752232
      A(6 ,3) = 19.51291278
      A(7 ,3) = 22.67158177
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      A(8 ,3) = 25.82603714
      A(9 ,3) = 28.97767277
      A(10,3) = 32.12732702
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,4) = 4.20118894
      A(2 ,4) = 8.01523660
      A(3 ,4) = 11.34592431
      A(4 ,4) = 14.58584829
      A(5 ,4) = 17.78874787
      A(6 ,4) = 20.97247694
      A(7 ,4) = 24.14489743
      A(8 ,4) = 27.31005793
      A(9 ,4) = 30.47026881
      A(10,4) = 33.62694918
C**** THESE ARE THE ROOTS OF THE DERIVATIVE OF THE BESSEL FUNCTION OF OR
C**** SET EQUAL TO ZERO
      A(1 ,5) = 5.31755313
      A(2 ,5) = 9.28239629
      A(3 ,5) = 12.68190844
      A(4 ,5) = 15.96410704
      A(5 ,5) = 19.19602880
      A(6 ,5) = 22.40103227
      A(7 ,5) = 25.58975968
      A(8 ,5) = 28.76783622
      A(9 ,5) = 31.93853934
      A(10,5) = 35.10391668
    1 BESPRIM = A(L, M)
      RETURN
      ENTRY BESSEL
C**** THESE ARE THE BESSEL NUMBERS OF ORDER ZERO  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,1) = 1.00000000
      A(2 ,1) = -0.4027588095
      A(3 ,1) = 0.301128303
      A(4 ,1) = -0.249704877
      A(5 ,1) = 0.218359407
      A(6 ,1) = -0.19645371
      A(7 ,1) = 0.180063375
      A(8 ,1) = -0.167184600
      A(9 ,1) = 0.156724985
      A(10,1) = -0.148011108
C**** THESE ARE THE BESSEL NUMBERS OF ORDER  ONE  FOR THE ZEROS OF THE B
```

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C**** FUNCTION
      A(1 ,2) = 0.5818649368
      A(2 ,2) = -0.3461258542
      A(3 ,2) = 0.2732981131
      A(4 ,2) = -0.233304416
      A(5 ,2) = 0.207012651
      A(6 ,2) = -0.188017488
      A(7 ,2) = 0.173459050
      A(8 ,2) = -0.161838211
      A(9 ,2) = 0.152282069
      A(10,2) = -0.144242905
C**** THESE ARE THE BESSEL NUMBERS OF ORDER  TWO  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,3) = 0.4864961885
      A(2 ,3) = -0.3135283099
      A(3 ,3) = 0.2547441235
      A(4 ,3) = -0.220881581
      A(5 ,3) = 0.197937434
      A(6 ,3) = -0.181010000
      A(7 ,3) = 0.167835534
      A(8 ,3) = -0.157195167
      A(9 ,3) = 0.148363778
      A(10,3) = -0.140878333
C**** THESE ARE THE BESSEL NUMBERS OF ORDER THREE FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,4) = 0.4343942763
      A(2 ,4) = -0.2911584413
      A(3 ,4) = 0.240738175
      A(4 ,4) = -0.210965204
      A(5 ,4) = 0.190419022
      A(6 ,4) = -0.175048405
      A(7 ,4) = 0.162954965
      A(8 ,4) = -0.153102409
      A(9 ,4) = 0.144866574
      A(10,4) = -0.137844513
C**** THESE ARE THE BESSEL NUMBERS OF ORDER FOUR  FOR THE ZEROS OF THE B
C**** FUNCTION
      A(1 ,5) = 0.3996514545
      A(2 ,5) = -0.2743809949
      A(3 ,5) = 0.229590468
      A(4 ,5) = -0.202763849
      A(5 ,5) = 0.184029896
      A(6 ,5) = -0.169878516
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      A(7 ,5) = 0.158655372
      A(8 ,5) = -0.149451156
      A(9 ,5) = 0.141714307
      A(10,5) = -0.135086328
      GO TO 1
      END
      SUBROUTINE CALCA4( A4, A5 )
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2,SOME,SOME1
      COMPLEX A4, A5, PSI, DUMMY, G1, G2, I
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, I, LHAT, MHAT, NHAT,
     1 PSI2 ,PI,X2,X1,SOME,SOME1
      A4 = AK1/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)/PSI(DUMMY)
      A5=BETAC*(G1(X2,NHAT)-G1(X1,NHAT))/EPSIKAT(LHAT,MHAT,NHAT,ALENGTH,
     1RLMDA)/PSI(DUMMY)
      RETURN
      END
      FUNCTION EPSIKAT(L,M,N,ALENGTH,RLMDA)
      DUM1 = ALENGTH
      IF( N    .NE. 0) DUM1 = DUM1/2.
      IF( M    .EQ. 0) GOTO 1
      EPSIKAT = (0.5-M    *M    /(2.*RLMDA*RLMDA))*DUM1
      RETURN
    1 EPSIKAT = DUM1/2.
      RETURN
      ENTRY EPSIZN
      IF( N .EQ. 0 ) GO TO 2
      EPSIKAT= ALENGTH / 2.
      RETURN
    2 EPSIKAT= ALENGTH
      RETURN
      ENTRY ETA
      EPSIKAT = (N-1)*(N-1) * 3.1415926 *3.1415926 / (ALENGTH * ALENGTH)
     1+RLMDA * RLMDA
      RETURN
      END
      FUNCTION EPSIRZN(L,M,N,ALENGTH,RLMDA)
      COMPLEX FNORM,DUM2
      DUM1 = 1.
      IF(M.EQ.0) DUM1 = .5
      DUM2=FNORM(L,M,N,RLMDA,ALENGTH)**2
      EPSIRZN=DUM1 *REAL(DUM2)/3.1415926
```

```
      RETURN
      END
      COMPLEX FUNCTION FNORM(L,M,N,RLMDA,ALENGTH)
      DUM1 = 3.1415926
      IF( M .EQ. 0 ) DUM1 = 6.2831852
      DUM1=DUM1*EPSIKAT(L,M,N,ALENGTH,RLMDA)*BESSEL(M+1,L)**2
      FNORM=CMPLX(DUM1,0.0)
      FNORM= CSQRT(FNORM)
      RETURN
      END
      COMPLEX FUNCTION G2(N)
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2, MU, W, F, I, SOME, SOME1
      COMPLEX DUM1, GDUM, BETA, GBUM, DUM
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, I, LHAT, MHAT, NHAT,
     1 PSI2 ,PI,X2,X1,SOME,SOME1
      GBUM(BETA,DUM1,N,AMACH,OMEGA,ALENGTH) = -(AMACH*BETA*BETA+2.*OMEGA
     1 *BETA)/(( N * 3.1415926 / ALENGTH )**2 - BETA**2 ) *(CEXP(DUM1*
     1 ALENGTH) * DUM1 * (-1.)**(N+2) - DUM1 )
      DUM(DUM1,ALENGTH,NHAT) = CEXP(DUM1*ALENGTH)*(-1.)**(NHAT+2)-1.
      BETA = BETA1
      DUM1 = CMPLX(-AIMAG(BETA1),REAL(BETA1))
      G2 = GBUM(BETA,DUM1,N,AMACH,OMEGA,ALENGTH)
      BETA = BETA2
      DUM1 = CMPLX(-AIMAG(BETA2),REAL(BETA2))
      G2 = G2 + C * GBUM(BETA,DUM1,N,AMACH,OMEGA,ALENGTH)
      IF( N .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) RETURN
      RETURN
      ENTRY PSI
      DUM1 = CMPLX( -AIMAG(BETA1), REAL(BETA1))
      G2  = DUM1*DUM(DUM1,ALENGTH,NHAT)/(NHAT**2*3.1415926**2/ALENGTH**2
     1 -BETA1**2)
      IF( NHAT .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) GO TO 3
      DUM1 = CMPLX( -AIMAG(BETA2), REAL(BETA2))
      G2  = ( G2 +C*DUM1*DUM(DUM1,ALENGTH,NHAT)/(NHAT**2*3.1415926**2/
     1 ALENGTH**2-BETA2**2))/EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)
      RETURN
    3 G2  = ( G2  + C * ALENGTH ) / EPSIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA)
      RETURN
      END
      SUBROUTINE CALMU(JP)
      COMPLEX AA,AB,AC,AD,AE,AF,AG,AH,AI,AJ,AK,AL,AM,AN,AO,A111
```

```
      COMPLEX A1,A2,A3,A4,A5,A6,A7,A8,A9,SUM1,SUM2,SUM3,SUM4,G1,G2,PSI
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2, MU, W, F, I, FNORM,D1,D2,AK1J,AK2J,SSG1,A11,SOME,SOME1
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, I, LHAT, MHAT, NHAT,
     1 PSI2 ,PI,X2,X1,SOME,SOME1
      COMMON / BLKB / MU( 3,30,29) , W(30)
      COMMON C11(30,30) , C10(30,30) , ERROR1 , ERROR2 , LDEX , NDEX
      D1=CEXP(I*ALENGTH*BETA1)
      D2=CEXP(I*ALENGTH*BETA2)*C
      A1=W(1)**2
      A4=OMEGA**2
      A2 = BETAC*BESSEL(MHAT+1,LHAT)/PSI2  /FNORM(LHAT,MHAT,NHAT,RLMDA,A
     1LENGTH)
      AK2J=I*GAMMA*(W( 1 )*(D1+D2)+AMACH*(BETA1*D1+BETA2*D2))
      AK1J=I*GAMMA*(W( 1 )*(1.+C)+AMACH*(BETA1+C*BETA2))
      DO 1 L = 1,LDEX
      RLMDB = BESPRIM(MHAT+1,L)
      A3 = BESSEL(MHAT+1,L)*CMPLX(1.0,0.0)
      DO 1 N = 1,NDEX
      N1 = N-1
      AI=(A4 - ETA(L,MHAT,N,ALENGTH,RLMDB))
      AH=(A1 - ETA(L,MHAT,N,ALENGTH,RLMDB))
      IF(L.EQ.LHAT.AND.N1.EQ.NHAT) GOTO 1
      AO=OMEGA
      OMEGA=W(1)
      AF=G1(X2,N1) - G1(X1,N1)
      AG=G2(N1)
      OMEGA=AO
      AN=CMPLX(0.0,0.0)
      IF(L.NE.LHAT) GOTO 31
      AN=BETAI*AK1J + BETAN*AK2J*(-1.)**N1 + AMACH*AG
      AN=AN*(A4 -A1)/AH
      AN=AN + I*(W(1) - OMEGA)*(GAMMA*(BETAI*(1. + C) + BETAN*(D1 +D2)
     1*(-1.)**N1) + AMACH*SSG1(ALENGTH,N1))
      AN=AN/PSI2/FNORM(LHAT,MHAT,NHAT,RLMDA,ALENGTH)/EPSIZN(LHAT,MHAT,
     1N1,ALENGTH,RLMDA)/AI
   31 CONTINUE
      MU(L,N,1)=(A2/A3)*AF/AI/EPSIKAT(L,MHAT,N1,ALENGTH,RLMDB) + AN
    1 CONTINUE
      NHAI = NHAT+1
      MU(LHAT,NHAI,1)=CMPLX(0.0 ,0.0 )
      DO 2 NX = 1,NDEX
```

```
      N = NX-1
      DO 2 NY = 1,NDEX
      NP = NY - 1
      IF(N.EQ.NP)3,4
    3 IF(NP.EQ.0)5,6
    4 C10(NX,NY)=GAMMA/2./(N**2-NP**2)* ALENGTH       /PI*((N+NP)*(SIN(
     1(N-NP)*PI*X2/ALENGTH)-SIN((N-NP)*PI*X1/ALENGTH))+(N-NP)*(SIN((N+NP
     1)*PI*X2/ALENGTH)-SIN((N+NP)*PI*X1/ALENGTH)))
      C11(NX,NY) = GAMMA / 2. / ( N**2 - NP**2) *  AMACH*NP*((NP+N)*(COS
     1((NP-N)*PI*X1/ALENGTH)-COS((NP-N)*PI*X2/ALENGTH))+(NP-N)*(COS((NP+
     1N)*PI*X1/ALENGTH)-COS((NP+N)*PI*X2/ALENGTH)))
      GO TO 2
    5 C10(NX,NY)=GAMMA*(X2-X1)
      C11(NX,NY)=0.0
      GO TO 2
    6 C10(NX,NY)=GAMMA*(ALENGTH/(4.*N*PI)*(SIN(2.*N*PI*X2/ALENGTH)
     1-SIN(2.*N*PI*X1/ALENGTH)) + (X2-X1)/2.0)
      C11(NX,NY) = - GAMMA*AMACH/2.*(SIN(N*PI*X2/ALENGTH)**2-SIN(N*PI*X1
     1/ALENGTH)**2)
    2 CONTINUE
      CALL W1(2)
      IF ( JP .EQ.1 )RETURN
      DO 7 J = 2,JP
      A1 = W(J) - W(J-1)
      IF (ABS(REAL(A1)/REAL(W(J-1))).LE.ERROR1.AND.ABS(AIMAG(A1)/AIMAG(W
     1(J-1))).LE.ERROR2) GO TO 13
      A1=W( J )**2
      A11=OMEGA**2
      A2=BETAC*BESSEL(MHAT+1,LHAT)/PSI2   /FNORM(LHAT,MHAT,NHAT,RLMDA,ALE
     1NGTH)
      A3 = BETAI*GAMMA*I*W( J )
      A4 = BETAN*GAMMA*I*W(J  )
      A5 = 2.*I*W(J  )*AMACH
      AK1J=I*GAMMA*(W(J  )*(1.+C)+AMACH*(BETA1+C*BETA2))
      AK2J=I*GAMMA*(W(J  )*(D1+D2)+AMACH*(BETA1*D1+BETA2*D2))
      A6 = (AMACH*PI)**2/2./ALENGTH*CMPLX(1.0,0.0)
      DO 8 L = 1,LDEX
      RLMDB = BESPRIM(MHAT+1,L)
      A7 = CMPLX(BESSEL(MHAT+1,L) , 0.0 )
      DO 8 N = 1,NDEX
      N1 = N-1
      IF ( L.EQ.LHAT .AND. N1 .EQ. NHAT) GO TO 8
      SUM1 = SUM2 = SUM3 = SUM4 = CMPLX(0.0,0.0)
```

```
      DO 9 LP = 1,LDEX
      A8  = BESSEL(MHAT+1,LP)*CMPLX(1.0,0.0)
      DO 10 NP = 1,NDEX
      NP1=NP-1
   10 SUM3=SUM3+MU(LP,NP,J-1)*(C10(N,NP)*I*W(J  )+C11(N,NP))*A8
    9 CONTINUE
      DO 11 NP = 1,NDEX
      A8=MU( L   ,NP,J-1)
      SUM1 = SUM1+A8
      SUM2=SUM2+A8*(-1.)**(NP+1)
      IF(N.EQ.NP)11,12
   12 SUM4=SUM4+A8*(NP-1)**2*(1.-(-1.)**(NP+N))/((NP-1)**2-N1**2)
   11 CONTINUE
      A8 = EPSIKAT(L    ,MHAT,N-1,ALENGTH,RLMDB)*CMPLX(1.0,0.0)
      A9=EPSIZN(LHAT,MHAT,N-1,ALENGTH,RLMDA)*CMPLX(1.0,0.0)
      AI=OMEGA
      OMEGA=W(J  )
      AJ=G2(N1)
      AA = G1(X2,N1)-G1(X1,N1)
      OMEGA=AI
      AB = A2*AA/A8/A7
      AC = A3*SUM1/A9
      AD = A4*(-1.0)**(N+1)*SUM2/A9
      AE = BETAC/A8/A7*SUM3
      AF = -A5/A9*SUM4
      AG = -A6/A9*N1**2*MU(L,N,J-1)
      AI= A11-ETA(L,M,N,ALENGTH,BESPRIM(MHAT+1,L))
      AH=A1  -ETA(L,M,N,ALENGTH,BESPRIM(MHAT+1,L))
      AM=(AB+AC+AD+AE+AF+AG)/AH
      AN= CMPLX(0.0,0.0)
      IF(L.NE.LHAT) GOTO 400
      AN=(BETAI*AK1J + BETAN*AK2J*(-1.)**(N+1)+ AMACH*AJ)/PSI2/FNORM(
     1LHAT,MHAT,NHAT,RLMDA,ALENGTH)/A9/(AH*AI)
      AN=AN*(A11-A1)
      AO=SSG1(ALENGTH,N1)
      AN=AN+(W(J  )-OMEGA)*(I*GAMMA*BETAI*(1.+C) + I*GAMMA*BETAN*(D1+D2)
     1*(-1.)**N1+ 2.*I*AMACH*AO)/PSI2/FNORM(LHAT,MHAT,NHAT,RLMDA,ALENGTH
     2)/A9/AI
  400 CONTINUE
    8 MU(L,N,J)= AM+AN
      N1 = NHAT+1
      MU(LHAT,N1,J)=CMPLX(0.0,0.0)
    7 CALL W1(J+1)
```

```
      RETURN
   13 JP = J - 1
      RETURN
      END
      SUBROUTINE W1(J)
      COMPLEX SUM1,SUM2,SUM3,SUM4,A1,WIETA,G1,G2,PSI,FNORM,A,DUM1,DUM2
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1 AK2, PSI2, MU, W, F, I , BETA,SOME,SOME1,SUM5,DDUM,E,G
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, I, LHAT, MHAT, NHAT,
     1 PSI2 ,PI,X2,X1,SOME,SOME1
      COMMON / BLKB / MU( 3,30,29) , W(30)
      COMMON C11(30,30) , C10(30,30) , ERROR1 , ERROR2 , LDEX , NDEX
      SUM1 = SUM2 = SUM3 = SUM4 = SUM5=CMPLX(0.0,0.0)
      DO 1 L = 1,LDEX
      A1=CMPLX(BESSEL(MHAT+1,L) , 0.0)
      DO 1 N = 1,NDEX
      E=MU(L,N,J-1)*A1*C10(NHAT+1,N)
      SUM5=E + SUM5
      SUM4=E*I*W(J-1) + MU(L,N,J-1)*A1*C11(NHAT +1,N) + SUM4
    1 CONTINUE
      DO 2 N1 = 1,NDEX
      N = N1 - 1
      IF(N.EQ.NHAT)GO TO 2
      A1 = MU(LHAT,N1,J-1)
      SUM1 = SUM1+A1
      SUM2 = SUM2 + A1*(-1.)**N
      SUM3 = SUM3+A1*N**2/(N**2-NHAT**2)*(1.-(-1.)**(N+NHAT))
    2 CONTINUE
      BETA=OMEGA
      OMEGA=W(J-1)
      WIETA=FNORM(LHAT,MHAT,NHAT,RLMDA,ALENGTH)*(I*OMEGA/EPSIZN(LHAT,MHA
     1T,NHAT,ALENGTH,RLMDA)*(GAMMA*(BETAI*SUM1+BETAN*(-1.)**(NHAT+2)*SUM
     12)-2.*AMACH*SUM3)+BESSEL(MHAT+1,LHAT)*BETAC/EPSIRZN(LHAT,MHAT,NHAT
     1,ALENGTH,RLMDA)*SUM4)
      OMEGA= BETA
      WIETA = WIETA+(W(1)**2 +(W(J-1)-W(1) )*SOME)
      DDUM=WIETA
      F    =CSQRT(WIETA)
      E=F
      IF(REAL(F    ).LT.0.0) F=-F
      G=(BESSEL(MHAT +1,LHAT)*BETAC/EPSIRZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA
     1)*SUM5+(GAMMA*BETAI*SUM1+GAMMA*BETAN*(-1.)**NHAT-AMACH*2.*SUM3)/EP
```

```
     1SIZN(LHAT,MHAT,NHAT,ALENGTH,RLMDA))*FNORM(LHAT,MHAT,NHAT,RLMDA,ALE
     1NGTH)
      G = G * I + SOME
      NN = 0
   20 CONTINUE
      WIETA = DDUM+(F-W(J-1))*G
      F = CSQRT(WIETA)
      IF( REAL(F) .LT. 0.0 ) F = -F
      IF ( CABS(F-E).LT.1.E-07 .OR.NN.GT.10)12,13
   13 E = F
      NN = NN+1
      GO TO 20
   12 W(J) = F
      WRITE(6,100)J, W(J)
      F = F / 1.84118378
      WRITE(6,101) F
      RETURN
  100 FORMAT('0 OMEGA(',I2,') = ',2G21.14)
  101 FORMAT('+'70X' W/W0 = '2G21.14)
      END
      COMPLEX FUNCTION G1 ( X , N )
      COMPLEX BETA1, BETA2, BETAN, BETAC, BETAI, BETAWI, OMEGA, C, AK1,
     1AK2,PSI2,MU,W,F,I,BETA,SOME,SOME1
      COMPLEX DUM1, GDUM, GBUM, DUM
      COMMON / BLKA / BETA1, BETA2, AMACH, GAMMA, OMEGA, ALENGTH, C, AK1
     1, AK2, BETAN, RLMDA, BETAI, BETAC, BETAWI, I, LHAT, MHAT, NHAT,
     1 PSI2 ,PI,X2,X1,SOME,SOME1
      GDUM(A,DUM1,BETA ,X)=1./(A**2-BETA**2
     1 )*(CEXP(DUM1*X)*(DUM1*COS(A*X)+A*SIN(A*X))-DUM1)
      IF( X .EQ. 0.0 ) GO TO 1
      A = N*3.1415926/ALENGTH
      G1=GDUM(A,I*BETA1,BETA1,X)*(OMEGA+AMACH*BETA1)
      IF( N .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) GO TO 2
      G1 = GAMMA * (G1 + C * GDUM(A,I*BETA1,BETA1,X)*(OMEGA+AMACH*BETA2
     1))*I
      RETURN
    1 G1 = CMPLX(0.0,0.0)
      RETURN
    2 BETA = C * OMEGA * X
      G1 = GAMMA * ( G1 + BETA)*I
      RETURN
      ENTRY SG1
      IF( X .EQ. 0.0 ) GO TO 11
```

```
      A = N*3.1415926/ALENGTH
      G1=GDUM(A,I*BETA1,BETA1,X)
      IF( N .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) GO TO 21
      G1 =           G1 + C * GDUM(A,I*BETA1,BETA1,X)
      RETURN
   11 G1=CMPLX(0.0,0.0)
      RETURN
   21 BETA=C*X
      G1=G1+BETA
      RETURN
      ENTRY SSG1
      IF( X .EQ. 0.0 ) GO TO 111
      A = N*3.1415926/ALENGTH
      G1=GDUM(A,I*BETA1,BETA1,X)*I*BETA1
      IF( N .EQ. 0 .AND. CABS( BETA2) .LE. 1.E-10 ) RETURN
      G1 =           G1 + C * GDUM(A,I*BETA1,BETA1,X)*I*BETA2
      RETURN
  111 G1=CMPLX(0.0,0.0)
      RETURN
      END
```

THE FOLLOWING IS AN EXAMPLE OF THE PUNCHED CARD INPUT.

```
ONE       1.984159  0.265865  9.07012   5181.661961.146      0.982
+7.61314123437926E-02+6.37493806192886E-02 1 0  N00.1000.0100.010     3    3C
```

```
THIS IS ENGINE NUMBER ONE
THE MACH NUMBER IS   .26586500000000
THE RATIO OF SPECIFIC HEATS IS   1.1460000000000
THE FREQUENCY IS   1.8080424719600     3.23641657590519E-14
THE LENGTH OF THE COMBUSTOR IS   1.9841590000000
THE PRIMARY MODE ASSUMED IS LHAT =  1 MHAT =  1 NHAT =  0
THE LINER STARTS AT  .0965 AND ENDS AT  .1035
A =   1.0000000000000     0.
BETAN =  1.16979672024435E-02 0.
G =   .91660000000000     0.
BETAC =   2.9539809563369      0.
K =   .29539809563370     0.
BETAWI =  7.61314123437886E-02 6.37493806193157E-02
N =   .58624670209981     -.23978101901084     1
BETAWID =  7.61314123437034E-02 6.37493806192015E-02
N =   .58624670210013     -.23978101901041     1
IMPEDANCE OF THE INJECTOR IS    .07613141234379    .06374938061929
N =   .58624670209980     -.23978101901073     1
OMEGA WIGGLEY =   1.8080424719600     3.23641657590519E-14
W/W0 WIGGLEY =   .98199999999997     1.75779116189323E-14
THE BACKING DISTANCE IS   .10000000000000
THE WIDTH OF THE APERATURE IS  6.92336549877837E-03
THE LENGTH OF THE APERATURE IS   .10077525063986
THE PEAK VELOCITY IS   .33852621759621     0.
THE DIMENSIONAL FREQUENCY IS   1.8080424719600
OMEGA( 1) =   1.8077193093524     1.91897240853318E-02
W/W0 =   .98182448106968     1.04224924713011E-02
OMEGA( 2) =   1.7957796526966     1.99017000807443E-02
W/W0 =   .97533970926934     1.08091871636758E-02
OMEGA( 3) =   1.7947392480612     1.90911623159963E-02
W/W0 =   .97477463551259     1.03689607324241E-02
OMEGA( 4) =   1.7937225329955     1.89437574447553E-02
W/W0 =   .97422242824421     1.02889008965499E-02
THE BACKING DISTANCE IS   .10000000000000
THE WIDTH OF THE APERATURE IS  6.92336549877837E-03
THE LENGTH OF THE APERATURE IS   .10081311485223
THE PEAK VELOCITY IS   .33852621759621     0.
THE DAMPING RATE IS  1.89437574447553E-02
THE LOG DAMPING RATE IS  129.86837748249 INVERSE SECONDS
THE EFFECTIVE DAMPING RATE IS  1.80389999333238E-02
THE EFFECTIVE L. D. R. IS   123.66583871121    INVERSE SECONDS
THE RADIUS IS   9.07012 INCHES OR    .75584 FEET
```

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